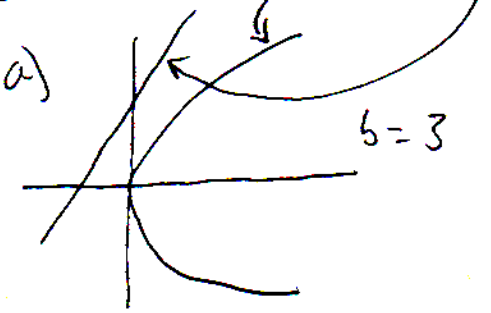
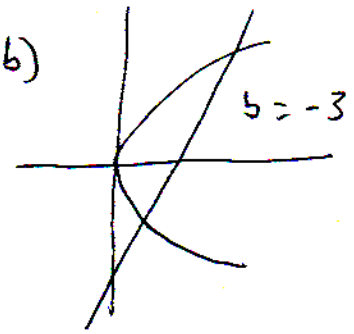
I

a) $\frac{1}{2}y^2 = 2x$ $\quad y - 6 = 2x$



b)

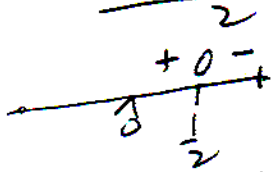


c) $\frac{1}{2}y^2 = y - 6$

$y^2 - 2y + 12 = 0$

$y = \frac{2 \pm \sqrt{4 - 86}}{2}$

$= \frac{2 \pm 2\sqrt{1-26}}{2}$

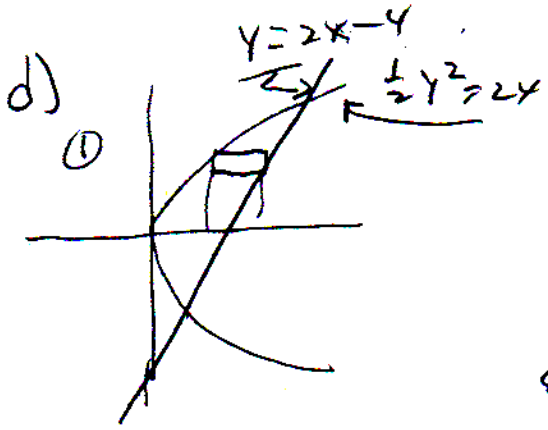


No solutions if $b > \frac{1}{2}$

1 sol if $b = \frac{1}{2}$

2 sol if $b < \frac{1}{2}$

d)



$\frac{1}{2}y^2 = y + 4$

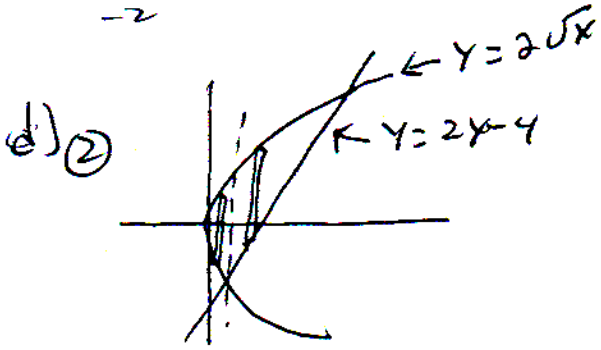
$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y \in \{4, -2\}$

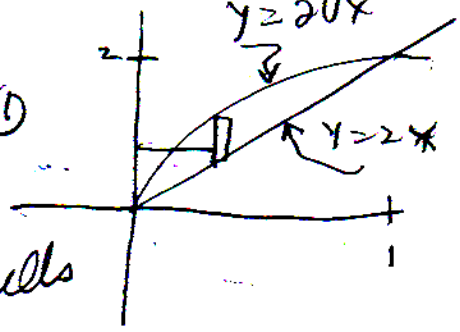
so $(1, -2)$ $(4, 4)$

$$\int_{-2}^{4} \frac{1}{2}y + 2 - \frac{1}{4}y^2 \, dy$$



$$2\int_0^1 2\sqrt{x}\,dx + \int_1^4 2\sqrt{x} - (2x-4)\,dx$$

e) ①



shells

$\frac{1}{2}y^2 = y$

$\frac{1}{2}y^2 - y = 0$

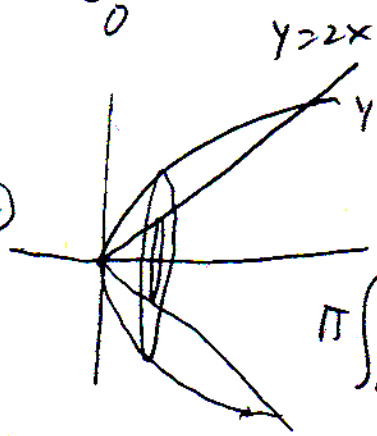
$y(\frac{1}{2}y - 1) = 0$

$y = 0 \quad y = 2$

$(0,0)$ $(1,2)$

$$2\pi\int_0^1 x(2\sqrt{x} - 2x)\,dx$$

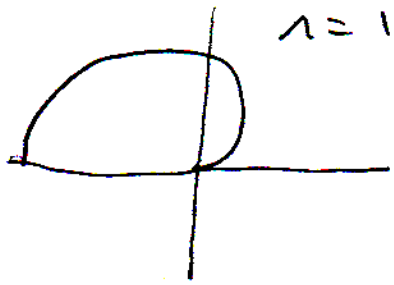
e) ②



washers

$$\pi\int_0^1 4x - 4x^2\,dx$$

II a)



$r = 1-\cos\theta \quad \dot{r} = \sin\theta$

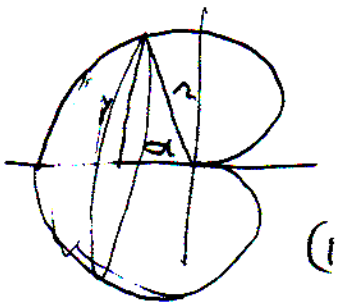
a) length $\int_0^{\pi} \sqrt{r^2+\dot{r}^2}\, d\theta$

$\int_0^{\pi} \sqrt{(1-\cos\theta)^2+\sin^2\theta}\, d\theta$

b) area

$\int \frac{r^2}{2}\, d\theta \qquad \frac{1}{2}\int_0^{\pi} (1-\cos\theta)^2\, d\theta.$

c)

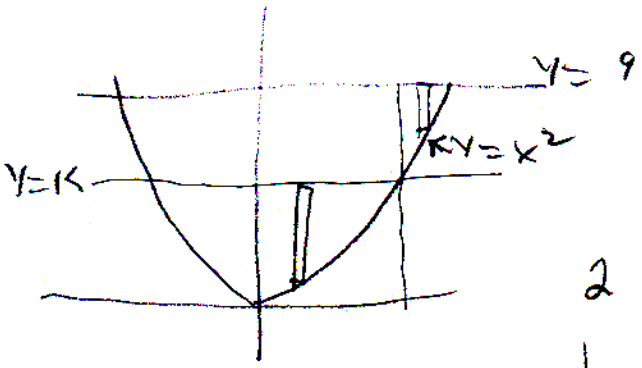


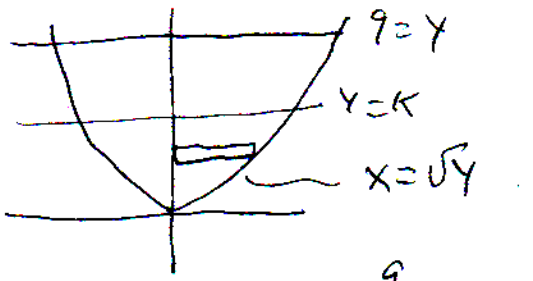
r is radius of rev which is $r\sin\theta$. which is $(1-\cos\theta)\sin\theta$.

$2\pi \int$ radius length $d\theta$

$2\pi \int_0^{\pi} (1-\cos\theta)\sin\theta \sqrt{(1-\cos\theta)^2+\sin^2\theta}\, d\theta$

Extra.





$2\int_0^k \sqrt{y}\, dy = 2\int_k^9 \sqrt{y}\, dy$

$\frac{2}{3}y^{\frac{3}{2}}\Big|_0^k = \frac{2}{3}y^{\frac{3}{2}}\Big|_k^9$

$k^{\frac{3}{2}} = 27 - k^{\frac{3}{2}}$

$2k^{\frac{3}{2}} = 27$

$k^{\frac{3}{2}} = \frac{27}{2}$

$k = \left(\frac{27}{2}\right)^{\frac{2}{3}}$

$\frac{1}{2}$ whole area $\overset{\text{set}}{=}$ area below $y=k$

$2\,\frac{1}{2}\int_0^3 9 - x^2\, dx \overset{\text{set}}{=} 2\int_0^{\sqrt{k}} k - x^2\, dx$

$9x - \frac{x^3}{3}\Big|_0^3 = 2\left(kx - \frac{x^3}{3}\right)\Big|_0^{\sqrt{k}}$

or $27 - 9 = 18$

$9 = k\sqrt{k} - \frac{k\sqrt{k}}{3}$

$-9 = \frac{2k^{\frac{3}{2}}}{3}$

$\frac{27}{2} = k^{\frac{3}{2}}$

$k = \left(\frac{27}{2}\right)^{\frac{2}{3}}$

100	0	98
90	3	
80	2	
70	5	
60	4	
59	6	
		42

1372

20

68.6

Advanced Placement Calculus

Name Calc Ch 10

Per ____ Date 4/13/2000

Copy original problem.

Convince *me* that **you** understand the concept!

Last Regular Exam

I Given $\frac{1}{2}y^2 = 2x$ and $y - b = 2x$

a) Sketch both graphs such that the two graphs have no intersection. Identify a typical value for b. (5 pts)

b) Sketch both graphs such that the two graphs have 2 distinct intersections. Identify a typical value for b. (5 pts)

c) Compute the set of values for b such that there will be ... (10 pts tot)

1) no intersection.
2) exactly 1 intersection.
3) exactly 2 intersections.

d) In the case where $b = -4$, let R be the area trapped between the curves. (10 pts ea)

1) Set up but do not evaluate the integral which describes the area of R using horizontal sections.
2) Set up but do not evaluate the integral which describes the area of R using vertical sections.

e) Let $b = 0$. (15 pts tot)

1) Let R be the area in the first quadrant between the curves. Sketch R. Set up but do not evaluate the integral which describes the volume of the solid when R is revolved about the y-axis using the method of "shells".
2) Let R be the area in the first quadrant between the curves. Sketch R. Set up but do not evaluate the integral which describes the volume of the solid when R is revolved about the x-axis using the method of "washers".

II Given: $r = 1 - \cos\theta$ and $\theta \in [0, \pi]$. (15 pts ea)

1) Set up but do not evaluate the expression which describes the length of the curve.
2) Set up but do not evaluate the expression which describes the area contained by the curve and above the Cartesian x-axis.
3) Allow the curve to rotate about the Cartesian x-axis. Set up but do not evaluate the integral which yields **surface** area.

Extra Credit ---------- 5 pts ----------

Let R be the area trapped below the line $y = 9$ and above the curve $y = x^2$. The line $y = k$ divides R such that the part of R above $y = k$ is equal to the part of R below $y = k$. Find k. Justify.

I a) $\int_0^1 x\sqrt{1-x^4}\,dx$ let $u = x^2$, $\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int_0^1 \sqrt{1-u^2}\,du$

let $u = \sin\theta$, $du = \cos\theta\,d\theta$

$\frac{1}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta = \frac{1}{2}\cdot\frac{1}{2}\int_0^{\frac{\pi}{2}} 1+\cos 2\theta\,d\theta$

$\frac{1}{4}\left(\theta + \frac{1}{2}\sin 2\theta\Big|_0^{\frac{\pi}{2}}\right) = \frac{1}{4}\cdot\frac{\pi}{2} = \frac{\pi}{8}$

b) $\int_1^4 \sqrt{t}\ln t\,dt$

$u = \ln t$ $dv = \sqrt{t}\,dt$

$du = \frac{1}{t}dt$ $v = \frac{2}{3}t^{\frac{3}{2}}$

$= \frac{2}{3}t^{\frac{3}{2}}\ln t - \frac{2}{3}\int_1^4 t^{\frac{1}{2}}dt$

$\left(\frac{2}{3}t^{\frac{3}{2}}\ln t - \frac{4}{9}t^{\frac{3}{2}}\right)\Big|_1^4$

$\frac{16}{3}\cdot 2\ln 2 - \frac{32}{9} - \left(0 - \frac{4}{9}\right)$

$\frac{32}{3}\ln 2 - \frac{28}{9}$

$\frac{96\ln 2 - 28}{9}$

c) $\int_0^1 \ln x\,dx$

$\lim_{a\to 0^+}\int_a^1 \ln x\,dx =$

$u = \ln x$ $dv = dx$

$du = \frac{1}{x}dx$ $v = x$

$\lim_{a\to 0^+} x\ln x - \int_a^1 dx$

$\lim_{a\to 0^+}\left(x\ln x - x\right)\Big|_a^1$

$\lim_{a\to 0^+}\left(1\cdot 0 - 1 - (a\ln a - a)\right)$

$\lim_{a\to 0^+}\left(-1 - (0 - 0)\right)$

-1

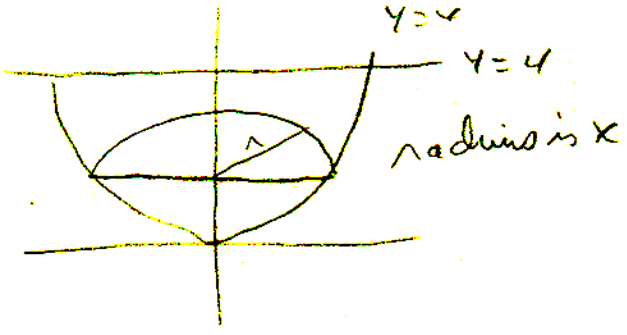
$\lim_{a\to 0^+} a\ln a =$

$\lim_{a\to 0^+} \frac{\ln a}{\frac{1}{a}}$ $\frac{-\infty}{\infty}$

$\lim_{a\to 0^+} \frac{\frac{1}{a}}{\frac{-1}{a^2}}$

$\lim_{a\to 0^+} -a = 0$

II



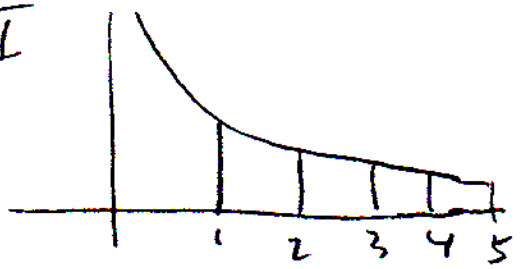
$\frac{\pi}{2}\int_{y=0}^{y=4} x^2\,dy =$

$\frac{\pi}{2}\int_0^4 y\,dy = \frac{\pi}{4}y^2\Big|_0^4 = 4\pi$

use dx instead of dy

-4

III



TRAPAZOID $\frac{h}{2}\left(f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right)$

$h=1$

$= \frac{1}{2}\left(1+2\cdot\frac{1}{2}+2\cdot\frac{1}{3}+2\cdot\frac{1}{4}+\frac{1}{5}\right)$

$= \frac{1}{2}\left(\frac{30+30+20+15+6}{30}\right)$

$= \frac{1}{2}\left(\frac{101}{30}\right) = \frac{101}{60}$

Simpsons $\frac{h}{3}\left(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+f(x_4)\right)$

$\frac{1}{3}\left(1+2\cdot\frac{1}{2}+2\cdot\frac{1}{3}+4\cdot\frac{1}{4}+\frac{1}{5}\right)$

$\frac{1}{3}\left(4+\frac{2}{3}+\frac{1}{5}\right)$

$\frac{1}{3}\left(\frac{60+10+3}{15}\right) = \frac{73}{45}$

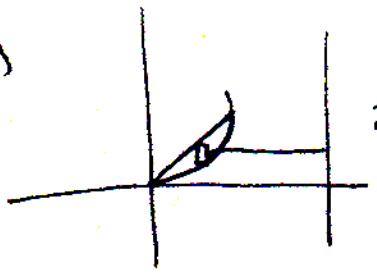
$\frac{101}{60}$ $\quad$ $\frac{73}{45}$

4.15 $\quad$ 3.15

$\frac{303-292}{180} = \frac{11}{180}$ This is the difference

IV

a)



shells

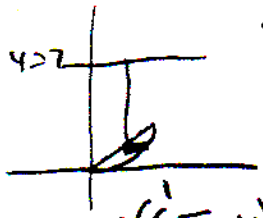
$2\pi\int_0^1 (x-x^2)(7-x)\,dx$

OR

washers

$\pi\int (7-y)^2-(7-\sqrt{y})^2\,dy$

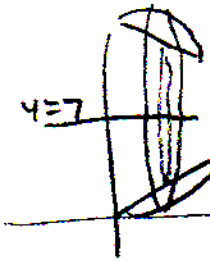
b)



shells

$2\pi\int_0^1 (\sqrt{y}-y)(7-y)\,dy$

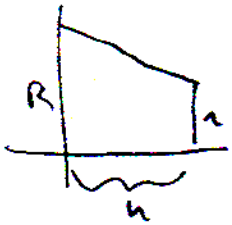
OR



washers

$\pi\int_0^1 (7-x^2)^2-(7-x)^2\,dx$

Extra



$y = mx+b$

$y = \frac{r-R}{h}x+R$

$y^2 = (mx+R)^2 = m^2x^2+2mRx+R^2$

$\pi\int_0^h m^2x^2+2mRx+R^2\,dx$

$\pi\left(\frac{m^2x^3}{3}+\frac{2mRx^2}{2}+R^2x\right)\Big|_0^h =$

$\pi\left(\frac{m^2h^3}{3}+mRh^2+R^2h\right)\Big|_{m=\frac{r-R}{h}}$

$\pi\left(\frac{(r^2-2rR+R^2)h^3}{3h^2}+\frac{(r-R)}{h}Rh^2+R^2h\right)$

$\pi\left(\frac{(r^2-2rR+R^2)h}{3}+rRh-\frac{R^2h+R^2h}{3}\right)$

$\frac{\pi}{3}\left(r^2h-2rRh+R^2h+3rRh-2R^2h\right)$

$\frac{\pi}{3}\left(r^2h+rRh+R^2h\right)$

$\frac{\pi h}{3}\left(r^2+rR+R^2\right)$

Imagine That!
Just like in the book formulas!

4/10/01

100	1
90	4
80	9
70	7
60	2
59	4

80.3

Great Exam!

It should be changed b/c cross sections are squares, so they can't rotate as well to get the answer.

Advanced Placement Calculus Name Calc Ch 10

Copy original problem. Per _____ Date 2001

Convince *me* that **you** understand the concept*!*

No Calculators !

Chapter 10 Exam

I Evaluate: (15 pts ea)

a) $\int_0^1 x\sqrt{1-x^4}\ dx$ b) $\int_1^4 \sqrt{t}\ \ln t\ dt$ c) $\int_o^1 \ln x\ dx$

II The base of solid S is the region $\{(x,y)|\ x^2 \le y \le 4\}$ (which means that $x^2 \le y$ *and* $y \le 4$), Cross sections perpendicular to the y-axis are semi-circles. Find the volume of S. (15 pts)

III Given the region bounded by the graphs of $x = 1$, $x = 5$, $y = 0$ and $y = \frac{1}{x}$. (20 pts)

Compare the approximations supplied by Simpson's Rule and the Trapazoidal Method when you use four sections..

IV Set up integras which represent the volumes of the following objects. Do not evaluate the integrals. (10 pts ea)

a) The solid obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the line $x = 7$.

b) The solid obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the line $y = 7$.

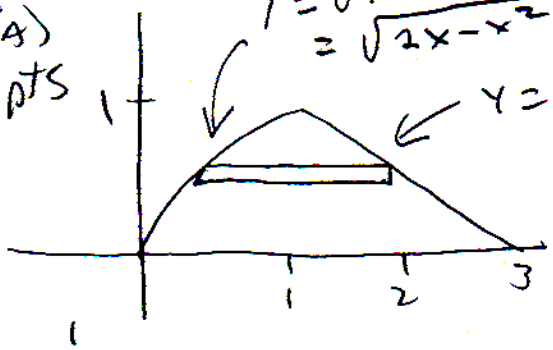
Extra Credit ---------- 5 pts ----------

There is a cone with the pointed end cut off leaving what is known as a frustum. The radius of the big end is R and the radius of the smaller end is r. The frustum has height h. On a standard cartesian axis mark the points (0 , R) and (h, r). Connect them with a straight line. When this figure is revolved about the x-axis you will have the frustum. Find the volume of the frustum.

In mathematics, it is not enough to read the words ... you *must* hear the music. – John Kelley

I

A) 10 pts



$2y = -x+3$

$x = 3-2y$

$(x-1)^2 + y^2 = 1$

$(x-1)^2 = 1-y^2$

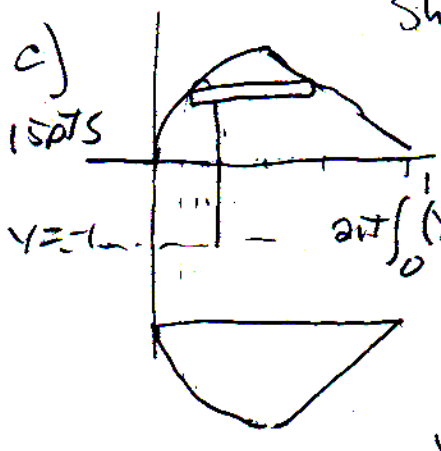
$x = 1+\sqrt{1-y^2}$

$$\int_0^1 3-2y-(1+\sqrt{1-y^2})\,dy$$

$$\int_0^1 2-2y-\sqrt{1-y^2}\,dy$$

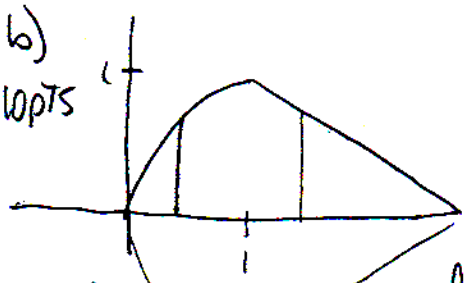
"Shells" using Part a multiply by 2π Radius

c) 15 pts



$$2\pi\int_0^1 (y+1)(2-2y-\sqrt{1-y^2})\,dy$$

b) 10 pts



Circle Part.

$y = \sqrt{1-(x-1)^2}$

$= \sqrt{2x-x^2}$

$y' = \frac{2-2x}{2\sqrt{2x-x^2}}$

$= \frac{1-x}{\sqrt{2x-x^2}}$

$(y')^2 = \frac{1-2x+x^2}{2x-x^2}$

$1+(y')^2 = \frac{2x-x^2+1+2x-x^2}{(2x-x^2)}$

$= \frac{1}{(2x-x^2)}$

$$2\pi\int_0^1 y\sqrt{\frac{1}{2x-x^2}}\,dx$$

$$2\pi\int_0^1 \sqrt{2x-x^2}\,\frac{1}{\sqrt{2x-x^2}}\,dx$$

$$2\pi\int_0^1 dx$$

Line Part.

$y = -\frac{1}{2}x+\frac{3}{2}$

$y' = -\frac{1}{2}$

$(y')^2 = \frac{1}{4}$

$1+(y')^2 = \frac{5}{4}$

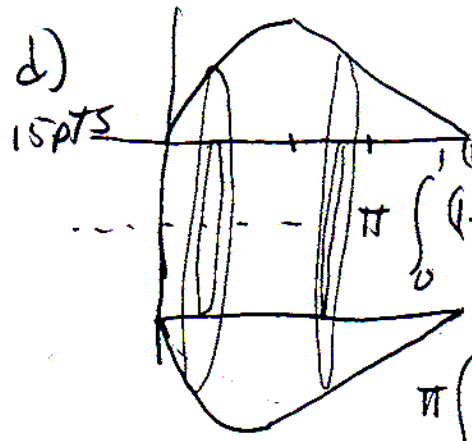
$$2\pi\int_1^3 \left(-\frac{1}{2}x+\frac{3}{2}\right)\cdot\sqrt{\frac{5}{4}}\,dx$$

$$\sqrt{5}\int_1^3 -\frac{1}{2}x+\frac{3}{2}\,dx$$

$$2\pi\int_0^1 dx + \frac{\sqrt{5}}{2}\int_1^3 -x+3\,dx$$

"washers"

d) 15 pts



$$\pi\int R^2-r^2\,dx$$

$$\pi\int_0^1 (1+\sqrt{2x-x^2})^2 - 1\,dx$$

$$+$$

$$\pi\int_1^3 \left(-\frac{1}{2}x+\frac{3}{2}+1\right)^2 - 1\,dx$$

$$\pi\int_0^1 2\sqrt{2x-x^2}+2x-x^2\,dx +$$

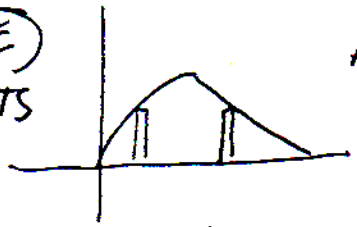
$$\pi\int_1^3 \frac{1}{4}x^2-\frac{5}{2}x+\frac{25}{4}-1\,dx$$

$$\pi\int_0^1 2\sqrt{2x-x^2}+2x-x^2\,dx +$$

$$\frac{\pi}{4}\int_1^3 x^2-10x+24\,dx$$

E) 10 pts

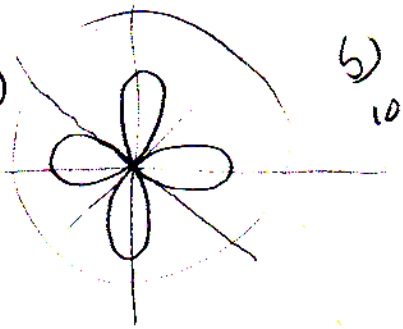
Average value



$$\frac{1}{3}\left[\int_0^1 \sqrt{2x-x^2}\,dx + \frac{1}{2}\int_1^3 3-x\,dx\right]$$

II

a) 5pts



b) 10pts $\int_{\theta=-\frac{\pi}{4}}^{\frac{\pi}{4}} d\theta = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{r^2+r'^2}\, d\theta$

$r = 3\cos 2\theta$

$r^2 = 9\cos^2 2\theta$

$r' = -6\sin 2\theta$

$r'^2 = 36\sin^2 2\theta$.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{9\cos^2 2\theta + 36\sin^2 2\theta}\, d\theta,$$

c) 10pts Area of $\frac{1}{2}$ of 1 leaf: $\int_0^{\frac{\pi}{4}} \frac{r^2\theta}{2} d\theta$, $= \frac{1}{2}\int_0^{\frac{\pi}{4}} 9\cos^2 2\theta = \frac{9}{2}\int_0^{\frac{\pi}{4}} \cos^2 2\theta\, d\theta$

There are 8 such $\frac{1}{2}$-leaves. So $36\int_0^{\frac{\pi}{4}} \cos^2 2\theta\, d\theta$

Area of circle $\pi(4)^2 = 16\pi$

$36\int_0^{\frac{\pi}{4}} \frac{1}{2}(1+\cos 4\theta)\, d\theta$

$18\int_0^{\frac{\pi}{4}} 1+\cos 4\theta\, d\theta = 18\left(\theta + \frac{1}{4}\sin 4\theta\right)\Big|_0^{\frac{\pi}{4}}$

$18\left(\frac{\pi}{4}+0\right) = \frac{9\pi}{2}$

$16\pi - \frac{9\pi}{2} = \frac{23\pi}{2}$

D) $16\pi \approx 50.27$, $36\int_0^{\frac{\pi}{4}} \cos^2 2\theta\, d\theta = 36(.128) = 4.608$ area inside circle, outside clover leaf.

e) Nothing to remark.

III 15pts total

$x(u) = \tan u \quad y(u) = \sec u$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{\frac{dy}{du}}{\frac{dx}{du}}$$

$$\frac{dy}{dx} = \frac{\sec u \tan u}{\sec^2 u}$$

$$\frac{dy}{du} = \frac{\tan u}{\sec u}$$

$$= \frac{\sin u}{\cos u}\cdot\frac{1}{\cos u}$$

$$\frac{dy}{dx} = \sin u$$

EXTRA

$\frac{d^2y}{dx^2}$ is 1st deriv of $\frac{dy}{dx}$

$$\therefore \frac{d^2y}{dx^2} = \cos u \cdot \frac{du}{dx}$$

$$= \cos u \cdot \frac{1}{\sec^2 u}$$

$$\frac{d^2y}{dx^2} = \cos^3 u$$

4/16/02

100	0
90	1
80	4
70	4
60	8
59	6

90

8

$\frac{1444}{22}$ 65.6

an excellent — EASY — EXAM.

Many students on crucial control. considering this year's situation many people lost at least one letter grade.

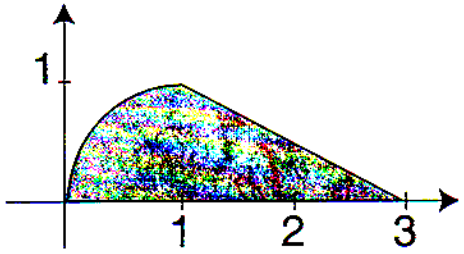
Advanced Placement Calculus Name Calc Ch 10

Copy original problem. Per _____ Date 2007

Convince *me* that you understand the concept!

Chapter 10 Exam

I R is the region shown shaded. The curves involved are:
$y = 0$, $2y = -x + 3$, and $(x-1)^2 + y^2 = 1$.
For each problem in this section, reproduce the picture at the right and indicate any significant items for that specific problem (such as the typical cross section).
Integrals are to be fully simplified but do not actually evaluate any of the integrals in this section.



a) Set up the explicit integral for the area of R using horizontal cross sections. (10 pts)

b) Set up the explicit which describes the surface area of the solid obtained when R is rotated about the x-axis. (10 pts)

c) Set up the explicit integral for the volume of the solid obtained by rotating R about the line $y = -1$. Use the "shell" method. (15 pts)

d) Set up the explicit integral for the volume of the solid obtained by rotating R about the line $y = -1$. Do **NOT** use the "shell" method. (15 pts)

e) Let $c(x)$ be the vertical cross section of R. Set up the explicit integral for the average value of $c(x)$. (10 pts)

II Given the two polar equations: $r = 4$ and $r = 3\cos 2\theta$ (25 pts tot)

a) Sketch both equations on the same polar axis.

b) Set up but do not actually evaluate the explicit integral for the arc length of one leaf of $r = 3\cos 2\theta$.

c) Find the area of the region inside the circle but outside of the clover leaf without using a calculator.

d) Find the area of the region inside the circle but outside of the clover leaf using a calculator.

e) Reconcile the differences in your answers to parts c and d above.

III Given: $x(u) = \tan u$ and $y(u) = \sec u$. (15 pts)

Find $\frac{dy}{dx}$ as functions of u. (fully simplified, of course.)

Extra Credit ---------- 5 pts ----------

Reconsider section III above. Find $\frac{d^2y}{dx^2}$ as functions of u. (fully simplified, of course.)

4/23/86

```
                0.  TC

          88.  +
3         86.  +
          80.  +
          79.  +
          79.  +
          78.  +
7         72.  +
          72.  +
          71.  +
          70.  +
          67.  +
          64.  +
4         62.  +
          60.  +
          56.  +
          56.  +
          56.  +
          55.  +
11        52.  +
          39.  +
          38.  +
          33.  +
          27.  +
          20.  +
           9.  +
       1 469.  T

       1 469.  ÷
25             oC

         25.   =
       58.76

       1 469.  ÷
         24.   =
61.2083333333
```

Excellent exam — "we are reminded of our mortality"

Now maybe we can cut the shit about B.S. and general time wasting — and prep me for the A.P.

used 1987 as take home test to be [illegible] without outside help (and signed and attested to) then corrected by the student.

worked very well.

Suggest swap III B, III C to suggest that II as a 4/4 is significant interval (if it creates one set of)

1989

100	0
90	2
80	3
70	1
60	6
59	9

95 ↓ 30

623

Students need Lots of Board work on this. Change this exam to a work sheet or take home exam ~~and then~~

Need to find similar types of problems for the exam.

areas each way
rotations etc.
Not good at this time of year.
Daniel give back points for corrections.
& force detailed solutions.

1990

90↑	7	98
80↑	5	↓
70↑	0	↓
60↑	1	65

89.08

There was board work.

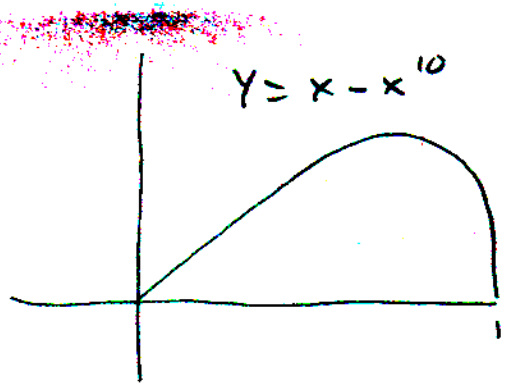
There were graded too easily. Besides it was posted & the detailed solutions were available so this is not a real fair comparison to last year's class.

Next year, we need to go over this test before they take Ch 10 exam — which will be different.

This one was SOO up because of the expected but canceled trip to Maryland.

I

4) $y = x - x^{10}$



B) $\int_0^1 x - x^{10}\,dx = \frac{x^2}{2} - \frac{x^{11}}{11}\Big|_0^1 = .409$

$\frac{1}{2} - \frac{1}{11} = \frac{9}{22}$

c)

$y = x - x^{10}$

$y' = 1 - 10x^9$

$(y')^2 = 1 - 20x^9 + 100x^{18}$

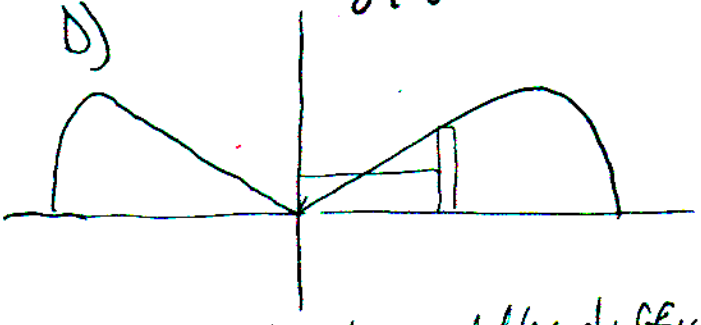
2pts $1 + \int_0^1 \sqrt{2 - 20x^9 + 100x^{18}}\,dx =$

1.8165

bottom of Region

2.8186

D)



using "washers" would be difficult because we would need to solve for x (in "this y value ...")

"Shells"

$2\pi \int_0^1 x(x - x^{10})\,dx$

$2\pi \int_0^1 x^2 - x^{11}\,dx$

$2\pi \left(\frac{x^3}{3} - \frac{x^{12}}{12}\right)\Big|_0^1 = \frac{1}{3} - \frac{1}{12}$

$2\pi\left(\frac{4-1}{12}\right)$

$\frac{3}{6}$

$\frac{1}{2}$

$\frac{\pi}{2}$

II

$y = f(x)$

a) $f' = 4x - 2$ (4,0)

$\int f' = \int 4x - 2\,dx$

$f = 2x^2 - 2x + c$

$f(1) = 2 - 2 + c = 0$ Sub

$c = 0$

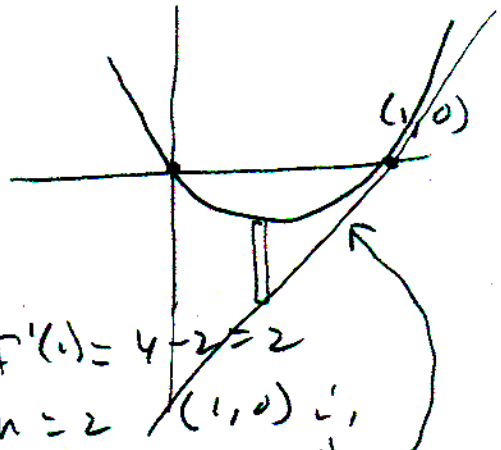
$f(x) = 2x^2 - 2x$

$2x(x-1)$

$f(0) = 0(-1) = 0$

f goes through (0,0)

b)



$f'(1) = 4 - 2 = 2$

$m = 2$ (1,0)

$y - 0 = 2(x-1)$

$y = 2x - 2$

c) $\int_0^1 2x^2 - 2x - (2x-2)\,dx$

$\int_0^1 2x^2 - 2x - 2x + 2\,dx$

$\int_0^1 2x^2 - 4x + 2\,dx$

$\frac{2}{3}x^3 - 2x^2 + 2x\Big|_0^1 =$

$\frac{2}{3} - 2 + 2$

$\frac{2}{3}$

III

a) $\frac{1}{2}(1-\cos 2\alpha) = \sin^2\alpha$

$\frac{1}{2}(1-(\cos^2\alpha - \sin^2\alpha) =$

$\frac{1}{2}(\sin^2\alpha + \sin^2\alpha)$

$\frac{1}{2}(2\sin^2\alpha) =$

$\sin^2\alpha = \sin^2\alpha$

b) Let $\beta = 2\alpha$

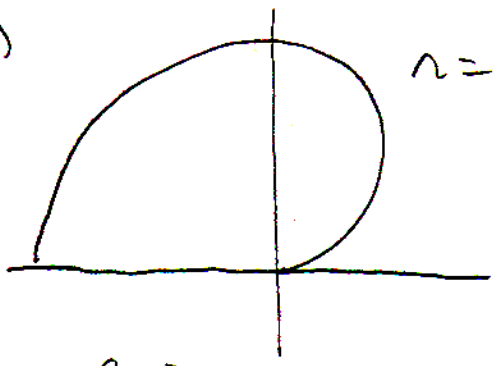
then

$\frac{1}{2}(1-\cos 2\alpha) = \sin^2\alpha$

becomes

$\frac{1}{2}(1-\cos\beta) = \sin^2\frac{\beta}{2}$

c)



$r = 1-\cos\theta$

$\theta \in [0, \pi]$

d) $r^2 = 1 + 2\cos\theta + \cos^2\theta$

$\dot{r} = -\sin\theta$

$\dot{r}^2 = \sin^2\theta$

$\int_0^{\pi} \sqrt{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}\, d\theta$

$2 - 2\cos\theta$

$\sqrt{2}\int_0^{\pi} \sqrt{1-\cos\theta}\, d\theta$

$\sqrt{2}\int_0^{\pi} \sqrt{2\left(\sin^2\frac{\theta}{2}\right)}\, d\theta$

$\sqrt{2}\sqrt{2}\int_0^{\pi} \sin\frac{\theta}{2}\, d\theta = 2\left(-2\cos\frac{\theta}{2}\right)\Big|_0^{\pi} =$

$0 + 4 = 4$

e) $\int \frac{r^2}{2}\, d\theta$

$\frac{1}{2}\int_0^{\pi} 1 - 2\cos\theta + \cos^2\theta\, d\theta.$

$\frac{1}{2}\int_0^{\pi} 1 - 2\cos\theta + \frac{1}{2}(1+\cos 2\theta)\, d\theta$

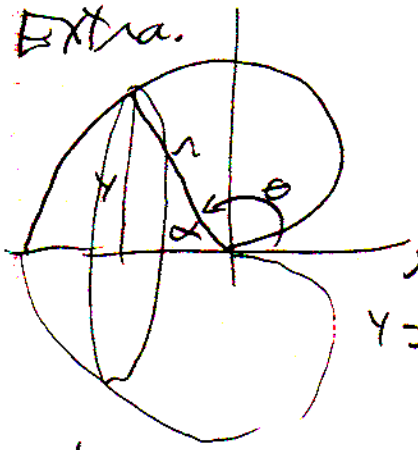
$\frac{1}{2}\int_0^{\pi} \frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos 2\theta\, d\theta$

$\frac{1}{2}\left(\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta\right)\Big|_0^{\pi}$

$\frac{1}{2}\left(\frac{3\pi}{2} - 0 + 0 - [0]\right) = \frac{3\pi}{4}$

2.356

Extra.



want surface area.

Y is radius of RV:

$X = r\sin\theta - 1$

$Y = \sin\theta - \sin\theta\cos\theta.$

$2\pi \int$ radius Length $d\theta$

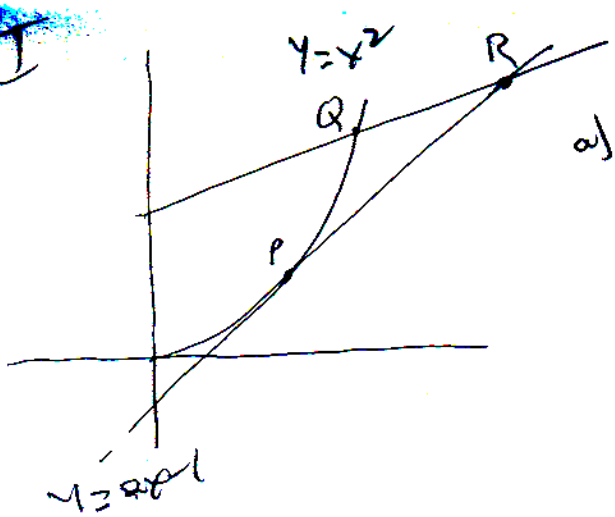
$2\pi \int_0^{\pi} (\sin\theta - \sin\theta\cos\theta)\sqrt{2}\sqrt{1-\cos\theta}\, d\theta$ (from Part d)

$2\sqrt{2}\pi \int_0^{\pi} \sin\theta\,(1-\cos\theta)^{\frac{3}{2}}\, d\theta.$

2.2627

20.106

7



a) $x+2 \overset{\text{Set}}{=} 2x-1$

$3 = x$

$R(3, 5)$

$x^2 \overset{\text{Set}}{=} 2x-1$

$x^2-2x+1=0$

$(x-1)^2=0$

$x=1$

$P(1, 1)$

$y = x+2$

$x+2 \overset{\text{Set}}{=} x^2$

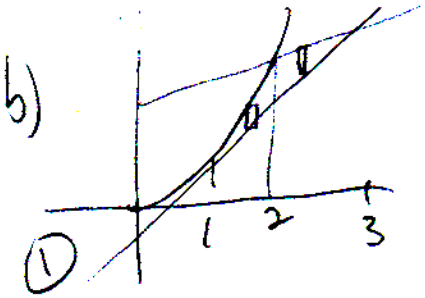
$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2$

$Q(2, 4)$

b)



①

$$\int_1^2 x^2-(2x-1)\,dx + \int_2^3 x+2-(2x-1)\,dx$$

$$\int_1^2 x^2-2x+1\,dx + \int_2^3 3-x\,dx$$

$$\frac{x^3}{3}-x^2+x\Big|_1^2 + 3x-\frac{x^2}{2}\Big|_2^3$$

$$\frac{8}{3}-4+2-\frac{1}{3}+1-1 + 9-\frac{9}{2}-6+2$$

$$\frac{7}{3}-2+5-\frac{9}{2}$$

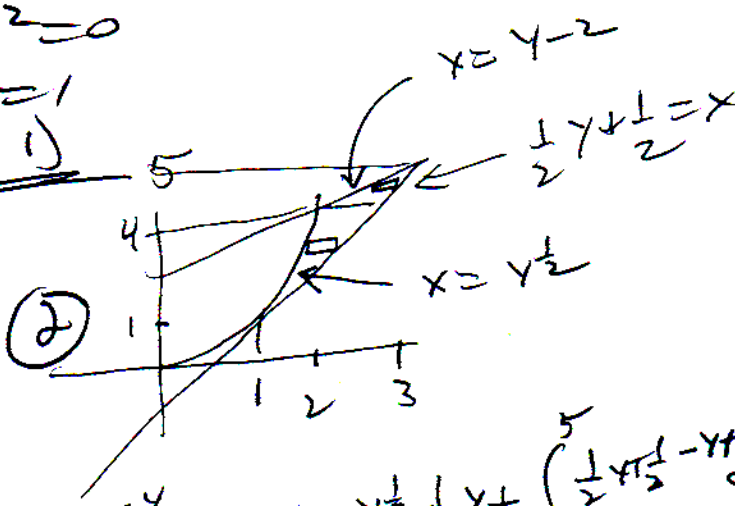
$$2\tfrac{1}{3} \quad \tfrac{1}{3}+\tfrac{1}{2}$$

$$\frac{5}{6}$$

$x = y-2$

$\frac{1}{2}y+\frac{1}{2} = x$

$x = y^{\frac{1}{2}}$



②

$$\int_1^4 \tfrac{1}{2}y+\tfrac{1}{2}-y^{\frac{1}{2}}\,dy + \int_4^5 \tfrac{1}{2}y+\tfrac{1}{2}-y+2\,dy$$

$$\frac{y^2}{4}+\frac{1}{2}y-\frac{2}{3}y^{\frac{3}{2}}\Big|_1^4 + \int_4^5 -\tfrac{1}{2}y+\tfrac{5}{2}\,dy$$

$$-\frac{y^2}{4}+\frac{5}{2}y\Big|_4^5$$

$$4+2-\frac{16}{3}-\frac{1}{4}-\frac{1}{2}+\frac{2}{3} - \frac{25}{4}+\frac{25}{2}+4-10$$

$$6-\frac{14}{3}-\frac{3}{4}+\frac{25}{4}-6$$

$$-\frac{14}{3}+\frac{22}{4} = -\frac{14}{6}+\frac{11}{2}$$

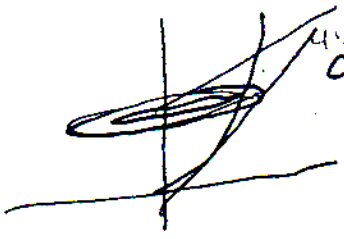
$$\frac{-28+33}{6} = \frac{5}{6}$$

c)

Same as part B with radius

$$2\pi\int_1^2 x^3-2x^2+x\,dx + 2\pi\int_2^3 3x-x^2\,dx$$

②



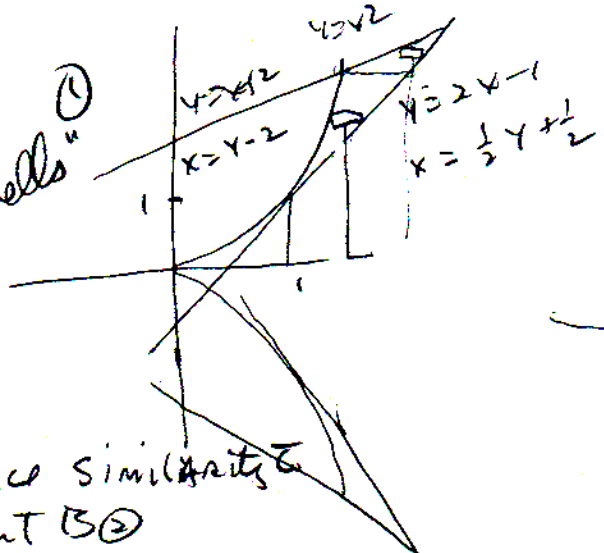
$$\pi\int_1^4 \left(\tfrac{1}{2}y+\tfrac{1}{2}\right)^2-\left(y^{\frac{1}{2}}\right)^2 dy + \pi\int_4^5 \left(\tfrac{1}{2}y+\tfrac{1}{2}\right)^2-(y-2)^2\,dy$$

$$\frac{\pi}{4}\int_1^4 y^2+2y+1-4y\,dy + \pi\int_4^5 \frac{y^2}{4}+\frac{y}{2}+\frac{1}{4}-y^2+4y-4\,dy$$

$$\frac{\pi}{4}\int_1^4 y^2-2y+1\,dy + \pi\int_4^5 -\frac{3y^2}{4}+\frac{9y}{2}-\frac{15}{4}\,dy$$

I

D ① "shells"



Notice similarity

Part B②

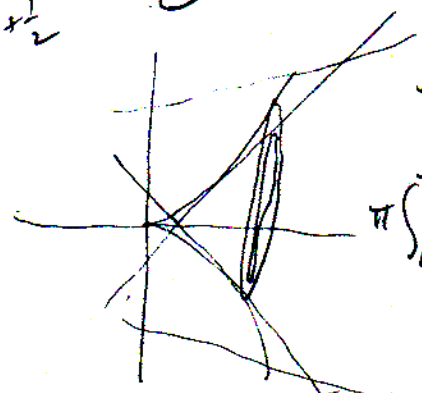
$$2\pi\int_1^4 y\left(\tfrac{1}{2}y+\tfrac{1}{2}-y^{\frac12}\right)dy+2\pi\int_4^5 y\left(-\tfrac{1}{2}y+\tfrac{5}{2}\right)dy$$

$$2\pi\int_1^4 \tfrac{1}{2}y^2+\tfrac{1}{2}y-y^{\frac32}\,dy+2\pi\int_4^5 -\tfrac{y^2}{2}+\tfrac{5y}{2}\,dy$$

11.6239 + 6.807

18.431

② "washes"



$$\pi\int_1^2 x^4-(2x-1)^2\,dx+\pi\int_2^3 (x+2)^2-(2x-1)^2\,dx$$

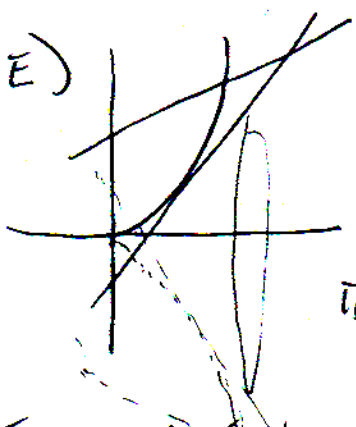
$$\pi\int_1^2 x^4-4x^2+4x-1\,dx+\pi\int_2^3 x^2+4x+4-4x^2+4x-1\,dx$$

Compute B①

$$\pi\int_1^2 x^4-4x^2+4x-1\,dx+\pi\int_2^3 -3x^2+8x+3\,dx$$

5.864 + 12.566

E)



$$2\pi\int_1^4 y\sqrt{1+\tfrac{1}{4y}}\,dy+2\pi\int_4^5 y\sqrt{1+\tfrac14}\,dy+2\pi\int_4^5 y\sqrt{1+1}\,dy$$

$$\pi\int_1^4 \frac{y\sqrt{4y+1}}{\sqrt{y}}\,dy+\pi\int_1^5 y\frac{\sqrt{5}}{2}\,dy+2\pi\int_4^5 y\sqrt{2}\,dy$$

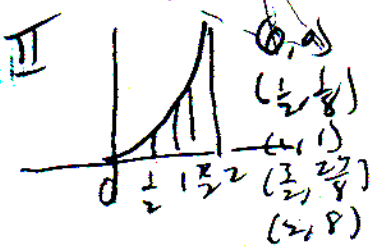
$$\pi\int_1^4 \frac{\sqrt{4y+1}}{\sqrt{y}}\,dy+\pi\sqrt{5}\int_1^5 y\,dy+2\pi\sqrt{2}\int_4^5 y\,dy$$

$\frac{dx}{dy}=\frac{1}{2y^{\frac12}}$ $\left(\frac{dx}{dy}\right)^2=\frac{1}{4y}$

$\frac{dx}{dy_2}=\frac12$ $\left(\frac{dy}{dx}\right)^2=\frac14$

$\frac{dx}{dy}=1$

II



a) $\frac{1}{2}\cdot\frac{1}{2}\left(0+2\cdot\frac{1}{8}+2\cdot1+2\cdot\frac{27}{8}+8\right)$

$\frac{1}{4}\left(\frac{1}{4}+2+\frac{27}{4}+8\right)=\frac{17}{4}$

b) $\frac{1}{2}\cdot\frac{1}{3}\left(0+4\cdot\frac{1}{8}+2\cdot1+4\cdot\frac{27}{8}+8\right)$

$\frac{1}{6}\left(\frac{1}{2}+2+\frac{27}{2}+8\right)=\frac{24}{6}=4$

III $x=1-\cos\theta$ $[0,\pi]$ $\dot{x}=\sin\theta$

a) length $\int_0^\pi\sqrt{x^2+\dot{x}^2}\,d\theta$

$\int_0^\pi\sqrt{1-2\cos\theta+\cos^2\theta+\sin^2\theta}\,d\theta$

$\int_0^\pi\sqrt{2-2\cos\theta}\,d\theta=\sqrt{2}\int_0^\pi\sqrt{1-\cos\theta}\,d\theta$

$=\sqrt{2}\cdot\sqrt{2}\int_0^\pi\sin\frac{\theta}{2}\,d\theta$

$2\left(-2\cos\frac{\theta}{2}\right)\Big|_0^\pi=-4(0-1)=4$ length

$\sin^2\frac{x}{2}=\frac{1}{2}(1-\cos x)$

$\sin^2\frac{\theta}{2}=\frac{1}{2}(1-\cos\theta)$

$\sqrt{2}\sin\frac{x}{2}=\sqrt{1-\cos x}$

area $\int\frac{x^2}{2}\,d\theta$

$\frac12\int_0^\pi(1-\cos\theta)^2\,d\theta:\frac12\int_0^\pi 1-2\cos\theta+\cos^2\theta\,d\theta=\frac12\int_0^\pi 1-\cos\theta+\frac12(1+\cos2\theta)\,d\theta=\frac12\int_0^\pi\frac{3}{2}-2\cos\theta+\frac12\cos2\theta\,d\theta=$

$\frac12\left[\frac{3}{2}\theta-2\sin\theta+\frac14\sin2\theta\right]_0^\pi=\frac12\left[\frac{3\pi}{2}-0+0-(0)\right]=\frac{3\pi}{4}$ area.

4/24/03

Bob Marcus 76
Jenna Fussell 75
Alan Savage 74
Dave Lund 70
Lauren Davis 66
Allison Ramis 66
Thomas Olson 60
Matt Fouts 60
Craig Abernathy 58
Kristin Jones 50
Lauren Takeda 45
Jacob Damont 45

Bond tour costs
goodes.

Yep.
are goode 62.1

Advanced Placement Calculus

Name Calc Ch 10 Mu

Per _____ Date 2003

Copy original problem.

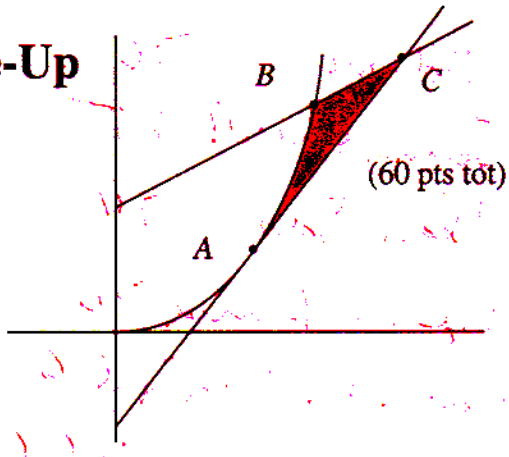
Convince *me* that **you** understand the concept!

No Calculators, of course.

Last Regular Exam Make-Up

I The region **R** is bounded by the curves . (60 pts tot)

$y = x^2$, $y = 2x - 1$, and $y = x + 2$. See picture →



a) Redraw the figure and determine coordinates of the intersection points *A*, *B*, and *C*.

b) Compute the area of **R**

1. using vertical cross sections.
2. using horizontal cross sections.

c) Set up a fully simplified integral expression **(but do not actually integrate and compute the numberical value)** for the volume when **R** is rotated about the *y*–axis

1. using "shells."
2. using "washers."

d) Set up a fully simplified integral expression **(but do not actually integrate and compute the numberical value)** for the volume when **R** is rotated about the *x*–axis.

1. using "shells."
2. using "washers."

e) Set up a fully simplified integral expression **(but do not actually integrate and compute the numberical value)** for the **surface area** generated when **R** is rotated about the *x*–axis.

II Given: $\int_0^2 x^3 \, dx$. Divide the interval into 4 equal sections and (20 pts tot)

a) use trapazoid method to approximate the definite integral.

b) use Simpsons's method to approximate the definite integral.

III Given: $r = 1 - \cos\theta$ for $\theta \varepsilon [0, \pi]$. (20 pts tot)

a) Determine the length of the curve.

b) Compute the area trapped below the curve and above the cartesian *x*-axis.

Band Tour took a bunch
and they "got" a bunch of customers.

	4	5		
100	0	2	2	103
90	4	2	6	
80	1	4	5	
70	3	2	5	
60	0	2	2	
54	0	1	1	

$\frac{666}{8}$ $\frac{1040}{13}$ 81.2

83.3 80

Advanced Placement Calculus

Name Calc ch10

Per _____ Date 2003

Copy original problem.

Convince *me* that **you** understand the concept!

Decimals must be accurate to 3 places!

Chapter 10 Exam

I Given $f(x) = x - x^{10}$ (30 pts tot)

a) Using the window: $x \in [-0.1, 1.1]$ and $y \in [-0.1, 1.1]$, sketch the graph of $f(x)$.

b) Let R be the region in the first quadrant under $f(x)$. Set up the integral which will describe the area contained in R. Evaluate the integral.

c) Set up an expression containing the appropriate integral equivalent to the ***perimeter*** of R. Your expressions must be fully simplified. Evaluate this expression using your grapher.

d) Let the region R be revolved about the y-axis. Briefly discuss the various options for finding the resulting volume. Which option would be the easiest to use? Explain. Use that option to find the volume. Do **not** use a calculator to evaluate the integrate.

II *Do **not** use your grapher in this section.* For the curve $y = f(x)$, it is given that $f'(x) = 4x - 2$ and the curve passes through the point (1, 0). (30 pts tot)

a) Determine the equation of the curve and show that it also passes through the origin.

b) Sketch the region bounded by this curve, the line $x = 0$ and its tangent at (1, 0). Find the equation of that tangent line,

c) Write an integral expression in terms of a single variable for the region described in part b above and then evaluate that integral.

III Given: $r = 1 - \cos\theta$ for $\theta \in [0, \pi]$ (40 pts tot)

a) Using appropriate trigonometric identities, prove that $\frac{1}{2}(1 - \cos 2\alpha) = \sin^2 \alpha$.

b) Using the results of part (a), argue the validity of: $\frac{1}{2}(1 - \cos\beta) = \sin^2 \frac{\beta}{2}$

c) Draw the graph of r.

d) Determine the length of the curve ***without*** using your calculator.

e) Find the area R trapped below the curve and above the Cartesian x-axis. (i.e.. In the 1st and 2nd Cartesian quadrants). Compute the area without using the calculator and then verify your work using the calculator.

Extra Credit ---------- 5 pts ----------

Continuing the problem from section III above, allow "r" to rotate about the Cartesian x-axis. Set up the integral which yields the surface area. Be sure your method is clear and justified. Evaluate your integral using your grapher.

EXAM CH 9 • • • • • • • • • • • • # 1 R/C Pg 362 - 366 • • • • • • • • • • • • # 2 Pg 366 1 - 3 3/17	# 3 Pg 366 4, 7, 8–13,16 also Draw cross sections # 4 R/C Pg 367 - 371 # 5 Pg 371 -372 1 - 5 # 6 Pg 372 9, 14 – 18 3/18	3/19	# 7 Pg 373 1 - 5, 8, 11, 12, 19 ***Also compute using grapher*** 3/20	# 8 Pg 374 21, 31 - 35 • • • • • • • • • • • • # 9 R/C Pg 375 - 379 3/21
# 10 Pg 379 1 - 8, 9 - 12, 19 ***Also compute using grapher*** ***Important Lecture*** 3/24	# 11 Pg 383 1 - 6, 9 Integrate answers to 9. Reconcicle differences • • • • • • • • • • • • # 12 "Voting Power" (an application of integrals) 3/25	# 13 Pg 391 1 - 17, 22, 27 • • • • • • • • • • • • # 14 R/C Pg 397 - 401 3/26	# 15 Pg 410 1 - 7 **Important Lecture** 3/27	# 16 Pg 410 9, 10, 12, 13 • • • • • • • • • • • • # 17 R/C pg 411-414 3/28
# 18 Pg 414 1–6,8,10, 14, 15 **do pblm 15 *using a single* integral** 3/31	# 19 Pg 420-421 1 - 8 **Important Lecture** 4 / 1	**Important Lecture** # 20 Pg 427 - 428 1, 2, 3, 19, 20, 25, 26 4 / 2	# 21 ReWrite ENTIRE "Summary" pg 429-432 # 22 Pg 421 22, 24, 25 GQ Pg 432 1, 3, 9 - 11 4 / 3	# 23, 24, 25 **"Obvious ...."** 4 / 4
# 26 Pg 432-436 1 - 4, 6 - 8, 25, 28, 30, 32, 4/7	# 27 Pg 432-436 39-41, 42a, 43 - 48 4/8	# 28 Pg 432-436 50, 51, 53, 54, 61 4/9	Put Last Year's Exam on the ChalkBoards 4/10	**Calculus Chapter 10** Exam 4/11
Begin AP prep work!!!! Easter Vacation 4/15	4/16	4/17	4/18	**2003**

I

a) $r = 3\sin\theta$

$r^2 = 9\sin^2\theta$

$\dot{r} = 3\cos\theta$

$\dot{r}^2 = 9\cos^2\theta$

$\int_0^{\pi} \sqrt{9\sin^2\theta + 9\cos^2\theta}\, d\theta$

$3\int_0^{\pi} d\theta$

b) $y = x^2 - \frac{1}{8}\ln x$

$y' = 2x - \frac{1}{8x}$

$(y')^2 = 4x^2 - \frac{1}{2} + \frac{1}{64x^2}$

$\int_1^4 \sqrt{(2x + \frac{1}{8x})^2}\, dx$

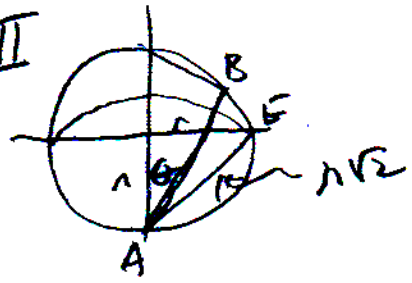
$\int_1^4 2x + \frac{1}{8x}\, dx$

$\int_1^4 \sqrt{1 + 4x^2 - \frac{1}{2} + \frac{1}{64x^2}}\, dx$

$\int_1^4 \sqrt{4x^2 + \frac{1}{2} + \frac{1}{64x^2}}\, dx$

$\frac{1}{2}\int r_1^2 - r_2^2\, d\theta$

II



$AE = r\sqrt{2}$

$\frac{AB}{2r} = \cos\theta$

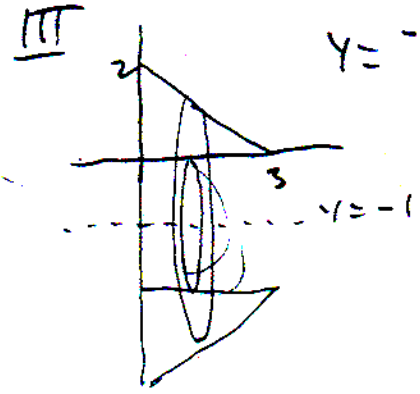
$AB = 2r\cos\theta$

$\frac{1}{2}\int_0^{\frac{\pi}{2}} 4r^2\cos^2\theta - 2r^2\, d\theta$

$r^2\int_0^{\frac{\pi}{2}} 2\cos^2\theta - 1\, d\theta = r^2\int_0^{\frac{\pi}{2}} 2(\frac{1}{2} + \frac{1}{2}\cos 2\theta) - 1\, d\theta$

$r^2\int_0^{\frac{\pi}{2}} 1 + \cos 2\theta - 1\, d\theta = r^2\int_0^{\frac{\pi}{2}} \cos 2\theta\, d\theta.$

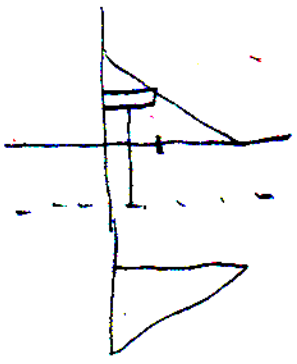
III



$y = -\frac{2}{3}x + 2$

a) $\pi\int_0^3 (-\frac{2}{3}x + 2 + 1)^2 - 1^2\, dx$

$\pi\int_0^3 \frac{4}{9}x^2 - 4x + 9 - 1\, dx = \pi\int_0^3 \frac{4}{9}x^2 - 4x + 8\, dx$



b) $2\pi\int_0^2 (y+1)(\frac{3}{2})(2-y)\, dy$

$3\pi\int_0^2 (y+1)(2-y)\, dy$

$3\pi\int_0^2 2 + y - y^2\, dy$

$y - 2 = -\frac{2}{3}x$

$\frac{2}{3}x = 2 - y$

$x = \frac{3}{2}(2-y)$

$y = 0 \quad x = 3$

$y = 2 \quad x = 0$

$dy = -\frac{2}{3}dx$

c) $2\pi\int_3^0 (-\frac{2}{3}x + 2 + 1) \times (-\frac{2}{3})\, dx = -\frac{4\pi}{3}\int_3^0 -\frac{2}{3}x^2 + 3x\, dx$

$\frac{4\pi}{3}\int_0^3 -\frac{2}{3}x^2 + 3x\, dx$

IV $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$

$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0$

$y^{\frac{2}{3}} = 1 - x^{\frac{2}{3}}$

$y = (1-x^{\frac{2}{3}})^{\frac{3}{2}}$

$\frac{1}{y^{\frac{1}{3}}}\frac{dy}{dx} = -\frac{1}{x^{\frac{1}{3}}}$

$\frac{dy}{dx} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$

$\left(\frac{dy}{dx}\right)^2 = \frac{y^{\frac{2}{3}}}{x^{\frac{2}{3}}}$

$1 + (\ )^2 = 1 + \frac{y^{\frac{2}{3}}}{x^{\frac{2}{3}}}$

$= \frac{x^{\frac{2}{3}} + y^{\frac{2}{3}}}{x^{\frac{2}{3}}}$

$= \frac{1}{x^{\frac{2}{3}}}$

Since $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ is original data.

$2\pi\int_0^1 (1-x^{\frac{2}{3}})^{\frac{3}{2}} \cdot \frac{1}{x^{\frac{1}{3}}}\,dx$

$x=0 \quad u=1$

$x=1 \quad u=0$

let $u = 1-x^{\frac{2}{3}}$

$du = -\frac{2}{3}x^{-\frac{1}{3}}dx$

$-\frac{3}{2}du = \frac{dx}{x^{\frac{1}{3}}}$

$\left(-\frac{3}{2}\right)2\pi\int_1^0 u^{\frac{3}{2}}\,du$

$3\pi\int_0^1 u^{\frac{3}{2}}\,du$

$3\pi\left(\frac{2u^{\frac{5}{2}}}{5}\right)\Big|_0^1 = \frac{6\pi}{5}$

Extra.

$\int \frac{x+2}{x^4-1}\,dx$

$(x-1)(x+1)(x^2+1)$

$\frac{x+2}{x^4-1} = \frac{a}{x+1} + \frac{b}{x-1} + \frac{cx+d}{x^2+1}$

$x+2 = a(x-1)(x^2+1) + b(x+1)(x^2+1) + (cx+d)(x^2-1)$

if $x=1$ $\quad 3 = 0 + 4b \quad \therefore b = \frac{3}{4}$

if $x=-1$ $\quad 1 = -4a + 0 + 0 \quad a = -\frac{1}{4}$

if $x=0$ $\quad 2 = -a + b - d$

$2 = \frac{1}{4} + \frac{3}{4} - d$

$d = -1$

if $x=2$ $\quad 4 = a(1)(5) + b(3)(5) + (2c+d)(3)$

$4 = -\frac{5}{4} + \frac{45}{4} + 6c - 3$

$7 = 10 + 6c$

$-3 = 6c$

$c = -\frac{1}{2}$

$-\frac{1}{4}\int\frac{dx}{x+1} + \frac{3}{4}\int\frac{dx}{x-1} - \frac{1}{2}\int\frac{x+2}{x^2+1}\,dx$

$-\frac{1}{4}\ln|x+1| + \frac{3}{4}\ln|x-1| - \frac{1}{2}\left[\frac{1}{2}\int\frac{2x}{x^2+1}\,dx + 2\int\frac{dx}{x^2+1}\right]$

$-\frac{1}{4}\ln|x+1| + \frac{3}{4}\ln|x-1| \rightsquigarrow -\frac{1}{4}\ln(x^2+1) - \tan^{-1}x$

$-\frac{1}{2}\,\frac{x-1}{-x-2}$

$\frac{2}{x+2}$

$-\frac{x+2}{2(x^2+1)}$

94
91 -2

87
86
86
85 -2
83
81
80 -2

76
73 -2
71

68
65
64 -2
62
61 -2

50
41

The attitude was "who cares?" when I asked about II (which was a portion of a problem) how do we do it in the method of the section. They just want final answers - what for? I have no idea.

Prob IV was ~~from~~ the prob.

No one finished early - but then I'm not convinced that they really tried.

$\frac{1858}{25} = 74.32$

Advanced Placement Calculus AB Name Calc Ch 10 1991

Copy the original problem.
Convince *me* that **you** understand the concept. Per _____ Date _____

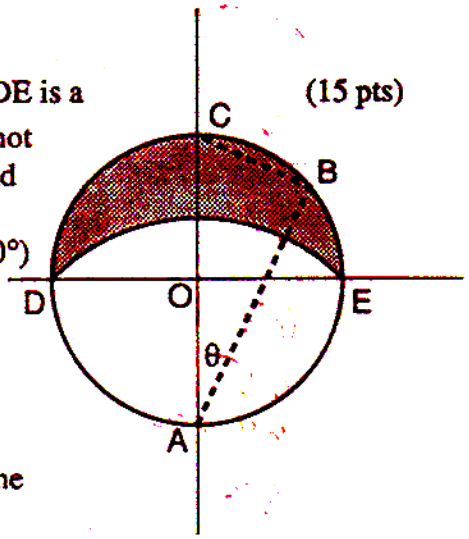
LAST REGULAR EXAM

Note: **"Set-up but do not integrate"** means to set-up the definite integral, including limits; clear constants across the integral sign; clear parantheses where possible and to fully simplify the integrand. You are finished with the set-up when the next step would be to integrate the expression.

I Set-up but do not integrate a **single integral** which finds the length of the curve ... (10 pts ea)

a) described by $r = 3\sin\theta$ for $\theta \in [0, \pi]$ b) described by $y = x^2 - \frac{1}{8}\ln x$ for $1 \le x \le 4$

II Consider the figure. Given a circle with center O and radius r. Arc DE is a portion of another circle which has center at point A. Set-up but do not integrate a **single integral** which finds the area trapped in the first and second quadrants, inside the circle and above the arc DE as shaded. Hint: Δ ABC is a right triangle since angle AOC is a diameter (ie 180°) and angle ABC is an inscribed angle which has the same arc. The "ray" which "sweeps out" the desired area is AB. (15 pts)



III Consider a line joining (0, 2) and (3, 0). The area below the line in the first quadrant will be called *R*. Set-up but do not integrate a **single integral** which finds ... (15 pts ea)

a) the volume when *R* is rotated about the line $y = -1$. Use "washers."

b) the volume when *R* is rotated about the line $y = -1$. Use "shells"; parameter *y*.

c) the volume when *R* is rotated about the line $y = -1$. Use "shells"; parameter *x*.

IV The portion of the curve $x^{\frac{1}{3}} + y^{\frac{1}{3}} = 1$ situated in the first quadrant is revolved around the x-axis. Find the area of the surface produced. (**note**: You must perform the integration.) (20 pts)

Extra Credit _____ 5 pts _____

Integrate: $\int \frac{x+2}{x^4 - 1}\, dx$

I $a(t) = 70 + 12t - 12t^2$

$v(t) = \int a(t) = \int 70 + 12t - 12t^2 \, dt$

$v(t) = 70t + 6t^2 - 4t^3 + c \quad v(0) = 0$

$\therefore c = 0$

$-2t(2t^2 + 3t - 35)$

$-2t(2t + 7)(t + 5)$

$t = \{0, -\frac{7}{2}, -5\}$ Reject $-\frac{7}{2}, -5$, $t > 0$

	0		+		0		−
v		0					
	0		5				

max at $t = 5$

$x(t) = \int v(t) \, dt = \int 70t + 6t^2 - 4t^3 \, dt$

$x(t) = 35t^2 + 2t^3 - t^4 + d \quad d = 0$

initially at rest

$x(t) = 35t^2 + 2t^3 - t^4$

$-t^2(t^2 - 2t - 35)$

$-t^2(t - 7)(t + 5)$

$-25(-2)(10$

$500.$

max displacement is 500.

c) $f'(1) = 4 - 2 = 2 \quad (1, 0)$

$y - 0 = 2(x - 1)$

$y = 2x - 2$

$\int_0^1 2x^2 - 2x - (2x - 2) \, dx$

$\int_0^1 2x^2 - 4x + 2 \, dx$

$\frac{2}{3}x^3 - 2x^2 + 2x \Big|_0^1 = \frac{2}{3} - 2 + 2 = \frac{2}{3}$

II $f'(x) = 4x - 2$

$(1, 0)$

a) $f(x) = \int f'(x) \, dx$

$= \int 4x - 2 \, dx$

$= 2x^2 - 2x + c$ sat

$f(1) = 2 - 2 + c = 0$

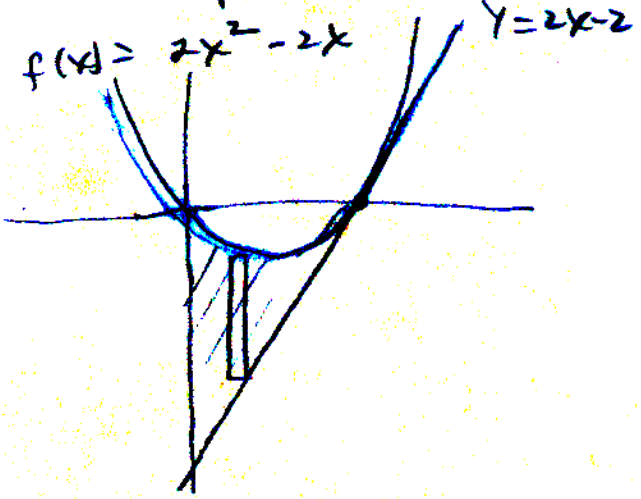
$c = 0$

$f(x) = 2x^2 - 2x$

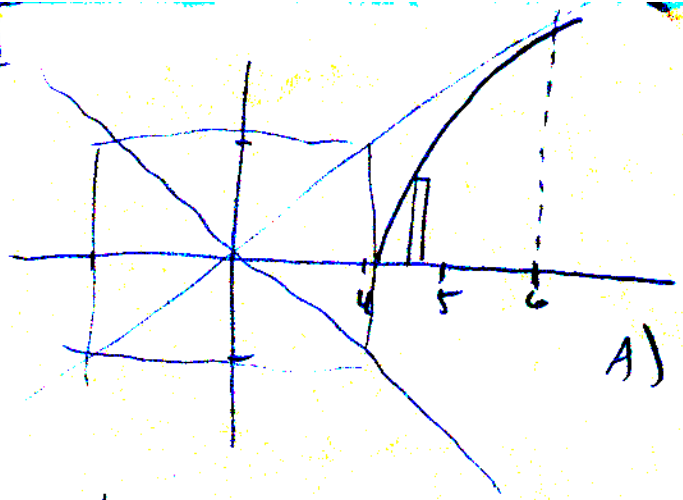
$2x(x - 1)$

$f(0) = 0 \therefore$ passes through origin.

b)



11



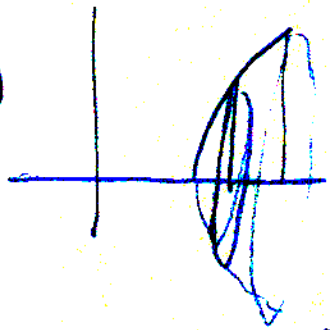
$x^2 - y^2 = 16$

$y^2 = x^2 - 16$

$y = +\sqrt{x^2-16}$

A) fnInt($\sqrt{x^2-16}$, x, 4, 6) = 5.717

b)



$\pi \int_4^6 x^2 - 16\, dx = 58.643$

$\pi *$ fnInt(x^2-16, x, 4, 6)

c)

$x^2 = 16 + y^2$

$x = \sqrt{16+y^2}$

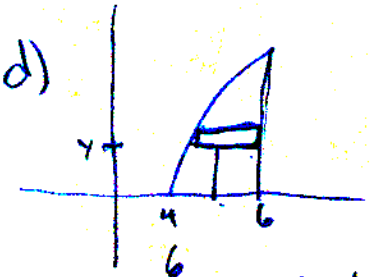
$36 - 16 = y^2$

$y = 2\sqrt{5}$

$2\pi \int_0^{2\sqrt{5}} y\,(6 - \sqrt{16+y^2})\, dy =$

58.643

d)



$x^2 - y^2 = 16$

$2x\,dx - 2y\,dy = 0$

$dy = \frac{x}{y}\,dx$

$2\pi \int_4^6 \sqrt{x^2-16}\,(6-x)\,\frac{x}{\sqrt{x^2-16}}\,dx$

2π fnInt($6x - x^2$, x, 4, 6) = 58.643

e) $y = \sqrt{x^2-16}$

$y' = \frac{2x}{2\sqrt{x^2-16}} =$

$y' = \frac{x}{\sqrt{x^2-16}}$

$1 + y'^2 = 1 + \frac{x^2}{x^2-16}$

$\frac{2x^2-16}{x^2-16}$

$2\pi \int_a^b x\sqrt{1+y'^2}\,dx$

$2\pi \int_4^6 \sqrt{x^2-16} \cdot \frac{\sqrt{2x^2-16}}{\sqrt{x^2-16}}\,dx$

$2\pi \int_4^6 \sqrt{2x^2-16}\,dx$

$2\pi *$ fnInt($\sqrt{2x^2-16}$, x, 4, 6)

72.915

Extra,

$y = k(k+1) \overset{\text{set}}{=} 2x^2$

$2x^2 - kx - k = 0$ has no solution when discriminant < 0 i.e. $b^2 - 4ac < 0$

$k^2 + 8k \overset{\text{set}}{=} 0$

$k(k+8) = 0$

+	0	−	0	+
	−8		0	

No solutions if $k \in (-8, 0)$ i.e. $-8 < k < 0$

100	0
90	2
80	2
70	3
60	5
59	3

94

401

$\frac{1049}{15} = 69.9$

The yearly walk of call!

Great exam!

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

***Any* decimals must be accurate to 3 places.**

Name Col 4th 4/4 Grade

Per _____ Date 4/26/95

Last Regular Exam

I A particle is initially at rest at the origin. The partical moves along the t - axis with acceleration $a(t) = 70 + 12t - 12t^2$ where time, $t > 0$. Find the maximum displacement from the origin. (25 pts)

II For the curve $y = f(x)$, it is given that $f'(x) = 4x - 2$ and the curve passes through the point (1, 0). DO NOT use a calculator for this section (25 pts tot)

a) Determine the equation of the curve and show that it also passes through the origin.

b) Sketch the region bounded by this curve, its tangent at (1, 0) and the ~~axis~~ line $x = 0$.

c) Find the equation of this tangent line, write an integral expression in terms of a single variable for the area described and then evaluate that integral. N I C

III Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 16$, the x - axis and the line $x = 6$. Four each section Sketch the approximating figure (10 pts ea)

a) Sketch R. Set up the integral expression in terms of a single variable for the area of R. Using your grapher, evaluate your integral.

b) In terms of the variable x, set up the integral expression in terms of a single variable for the volume of the solid obtained when the region R is revolved about the x - axis. **Do not use "shells."** Using your grapher, evaluate your integral.

c) Using "shells" in terms of the variable y, set up the integral expression in terms of a single variable for the volume of the solid obtained when the region R is revolved about the x - axis. Using your grapher, evaluate your integral.

d) Using "shells" in terms of the variable x, set up the integral expression in terms of a single variable for the volume of the solid obtained when the region R is revolved about the x - axis. Using your grapher, evaluate your integral.

e) In terms of the variable x, set up the integral expression in terms of a single variable for the **surface area** of the solid generated when the region R is revolved about the x - axis. Using your grapher, evaluate your integral.

Extra Credit ---------- 5 pts ----------

Given the line $y = k(x + 1)$ and the parabola $y = 2x^2$. For what values of k does the line have **no** intersection with the parabola?

~~83~~

90	2
80	5
70	6
60	4
59	3

93

15

~~only~~ corrections

every year....

How students do phlms Both as warm up and as skills for practice and to reduce longer extra corrections.

$\frac{1376}{20}$ 68.9

$\frac{1334}{18} = 74.1$ the 15 and dropping the 27.

Great exam!

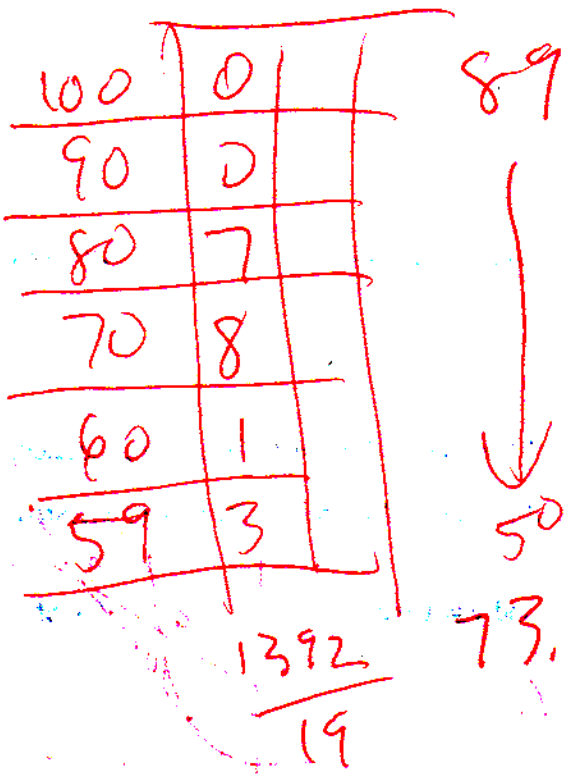
I want to do more with integrals to make it more interesting

I will require corrections — narratives & figs

~~omit mid-test~~ TC worth 10 HWs!

~~I~~ will compute #3. EXAM Grade + HW same ~~4 HW + 1~~ Grade.

96



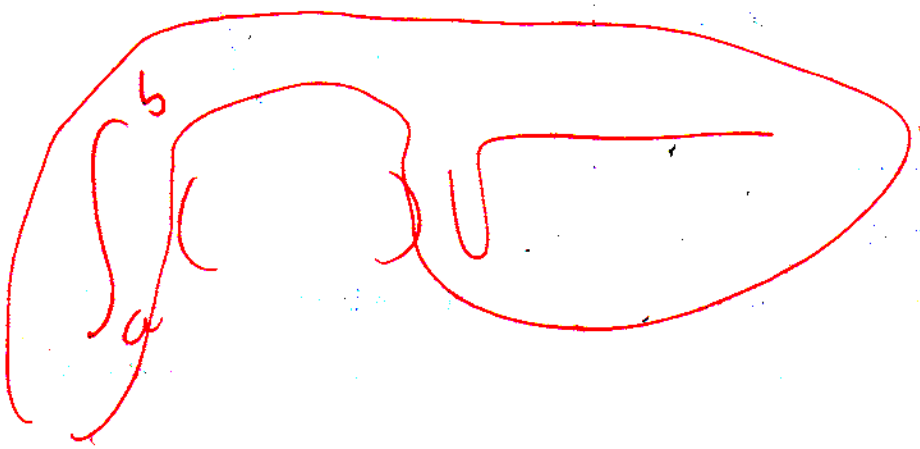
You play like you practice ...

Too bad For them!

make
change to $\frac{\pi}{2}$

either add
what is shown for f or swap it
with e to make similar but different

exam.
The latter choice might be best.
Surface area would be 3 integrals.



Give lengths of lines integrand ie. needs just length of line segment and axis

100	6
90	6
80	2
70	1
60	2
59	4

105

44

83.6

This is a very difficult
to grade exam. Takes
a long time. Need to be more specific about exactly
what is wanted ie "shells" about $x=-3$ instead
of y for example.

Advanced Placement Calculus

Name _____ 4th Quarter

Per _____ Date Calculus 4/23/97

Copy original problem.

Convince *me* that you understand the concept!

Last Regular Exam 4/9&10/97

Do not use calculators during this exam.

I Given the points (0 , – 5) and (2, 0). *Z* is the line joining these two points. (35 pts tot)

a) Compute the length of *Z* using the distance formula.

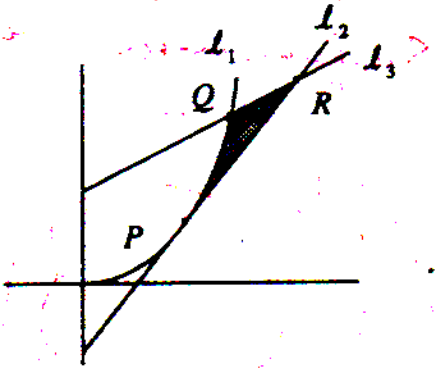
b) Compute the length of *Z* using the appropriate integral.

c) Allow *Z* to rotate about the *x*–axis. Compute the value of the integral for the surface area generated.

d) Allow *Z* to rotate about the *y*–axis. Compute the value of the integral for the surface area generated.

II The region **R** is bounded by the curves . (50 pts tot)

$y = x^2$, $y = 2x - 1$, and $y = x + 2$ See pict.



a) Determine coordinates of *P*, *Q*, *R*. Justify.

b) Identify l_1 , l_2, and l_3.

c) Compute the area of **R**

1. using vertical cross sections.
2. using horizontal cross sections.

d) Rotate **R** about the *y*–axis. Set up and simplify but **do not integrate** the integral which computes the resulting volume

1. using "shells."
2. using "washers."

e) Set up and simplify but **do not integrate** the integral which computes the resulting volume when **R** is rotated about the *x*–axis.

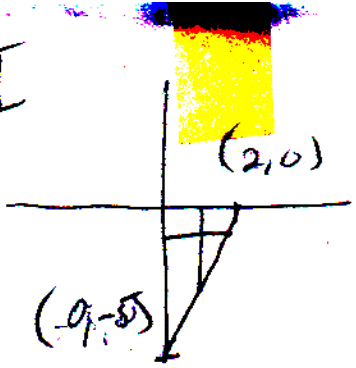
III Given: $\int_0^2 x^3\,dx$. Divide the interval into 4 equal sections and use Simpsons's method to approximate the definite integral. (15 pts)

Extra Credit ---------- 5 pts ----------

20 pts

An 8 by $2\sqrt{2}$ rectangle has the same center as a circle of radius 2. Find the area of the region common to both the rectangle and the circle.

I



a) $\sqrt{4+25}$

$\sqrt{29}$

b) $m = \frac{5}{2}$

$y = \frac{5}{2}x - 5$

$\int_0^2 \sqrt{1+\frac{25}{4}}\,dx$

$\frac{\sqrt{29}}{2}\int_0^2 dx$

$\frac{\sqrt{29}}{2}\left| x \right|_0^2 = \sqrt{29}$

c) radius $= -y$ 8 pts

$2\pi\int y\,ds$

$y = -\frac{5}{2}x + 5$

$2\pi\int_0^2 \left(-\frac{5}{2}x+5\right)\sqrt{1+\frac{25}{4}}\,dx$

$\pi\sqrt{29}\int_0^2 -\frac{5}{2}x+5\,dx$

$\pi\sqrt{29}\left(-\frac{5}{4}x^2+5x\Big|_0^2 \right.$

$-5+10$

$5\pi\sqrt{29}$

d) $2\pi\int x\,ds$

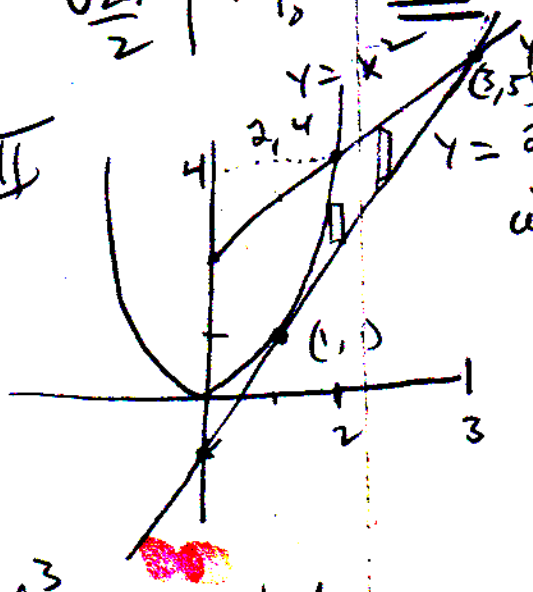
$2\pi\int_0^2 x\sqrt{1+\frac{25}{4}}\,dx$

$\pi\sqrt{29}\int_0^2 x\,dx$

$\pi\sqrt{29}\,\frac{x^2}{2}\Big|_0^2$

$2\pi\sqrt{29}$

III



a)

$x^2 \overset{\text{set}}{=} x+2$

$x^2-x-2=0$

$(x-2)(x+1)$

$x=2$

$x+2 \overset{\text{set}}{=} 2x-1$

$3 = x$

$x^2 \overset{\text{set}}{=} 2x-1$

$x^2-2x+1=0$

$(x-1)^2=0$

$x=1$

b)

① $\int_1^2 x^2-(2x-1)\,dx + \int_2^3 x+2-(2x-1)\,dx$

x^2-2x+1

$-x+3$

$\frac{x^3}{3}-x^2+x\Big|_1^2 + -\frac{x^2}{2}+3x\Big|_2^3$

$\frac{8}{3}-4+2 + -\frac{9}{2}+9+2-6$

$-\frac{1}{3}+1-1 \quad +\frac{9}{2}+\frac{12}{2}$

$\frac{7}{3}-\frac{6}{3} \quad \frac{1}{2}$

$\frac{1}{3} + \frac{5}{6}$ ✓

② $\int_1^4 \frac{1}{2}y+\frac{1}{2}-\sqrt{y}\,dy + \int_4^5 \frac{1}{2}y+\frac{1}{2}-(y-2)\,dy$

$\left(-\frac{1}{2}y+\frac{5}{2}\right)$

$\frac{y^2}{4}+\frac{1}{2}y-\frac{2}{3}y^{\frac{3}{2}}\Big|_1^4 + -\frac{1}{4}y^2+\frac{5}{2}y\Big|_4^5$

$4+2-\frac{16}{3}-\frac{1}{4}-\frac{1}{2}+\frac{2}{3} + -\frac{25}{4}+\frac{25}{2}+4-10$

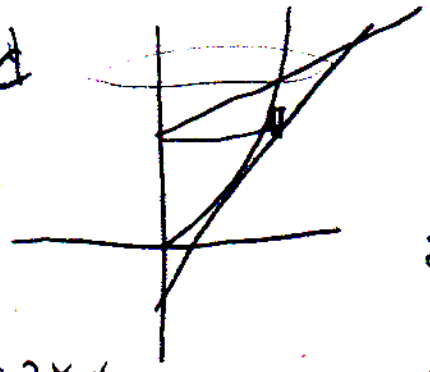
$6-\frac{14}{3}-\frac{3}{4} + \frac{25}{4}-6$

$-\frac{1}{2} \qquad \frac{1}{4}$

$\frac{36-28-3}{6}$

$\frac{5}{6}$ ✓

II d



$y = 2x-1$
$y = x+2$
$y = x^2$

① shells

$2\pi\int_1^2 x(x^2-(2x-1))\,dx + 2\pi\int_2^3 x((x+2)-(2x-1))\,dx$

$2\pi\int_1^2 x(x^2-2x+1)\,dx + 2\pi\int_2^3 x(-x+3)\,dx$

x^3-2x^2+x $\qquad -x^2+3x$

$2\pi\left(\frac{x^4}{4}-\frac{2}{3}x^3+\frac{x^2}{2}\Big|_1^2 + -\frac{1}{3}x^3+\frac{3}{2}x^2\Big|_2^3\right)$

$4-\frac{16}{3}+2-\frac{1}{4} + -9+\frac{27}{2}+\frac{8}{3}-6$

$+\frac{2}{3}\quad -\frac{1}{2}$

$\left(4-\frac{14}{3}-\frac{3}{4}\right) + \left(-15+\frac{35}{3}+\frac{51}{4}\right)$ $\Rightarrow$ $-9-2+\frac{51}{4}$

$\frac{-44+51}{4} = \frac{7}{4}\cdot 2\pi = \frac{7\pi}{2}$ ✓

② disks

$\pi\int_1^4 \left(\frac{1}{2}y+\frac{1}{2}\right)^2-(\sqrt{y})^2\,dy + \pi\int_4^5 \left(\frac{1}{2}y+\frac{1}{2}\right)^2-(y-2)^2\,dy$

$\frac{\pi}{4}\int_1^4 y^2+2y+1-4y\,dy + \pi\int \frac{1}{4}(y^2+2y+1)-y^2+4y-4\,dy$

$\frac{\pi}{4}\int_1^4 y^2-2y+1\,dy + \frac{\pi}{4}\int -y^2+2y+1 \;\; -4y^2+16y-16\,dy$

$\frac{\pi}{4}\left(\frac{y^3}{3}-y^2+y\Big|_1^4 + \frac{\pi}{4}\int -3y^2+18y-15\,dy\right.$

$\left.\frac{\pi}{4}\left(-y^3+9y^2-15y\Big|_4^5\right.\right.$

$\frac{64}{3}-16+4$ $\qquad \frac{\pi}{4}\left(-125+225-75+64-144+60\right)$

$-\frac{1}{3}+1-1$

$\frac{63}{3}$

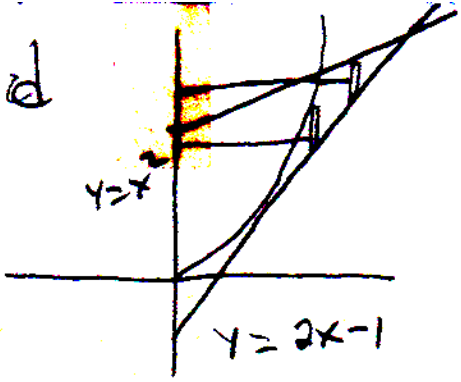
$21-16+4$

$\frac{9\pi}{4}$ $\qquad \frac{5\pi}{4}$

$+$

$\frac{14\pi}{4} = \frac{7\pi}{2}$ ✓

II d



$y=x+2$

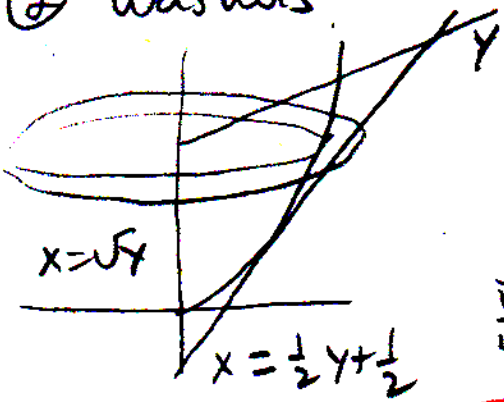
① shells

notice used II b ①
is 2π radius · II b ①

$$2\pi\int_1^2 x\left(x^2-(2x-1)\right)dx + 2\pi\int_2^3 x(x+2-(2x-1))\,dx$$

$$2\pi\int_1^2 x^3-2x^2+x\,dx + 2\pi\int_2^3 -x^2+3x\,dx$$

② "washers"

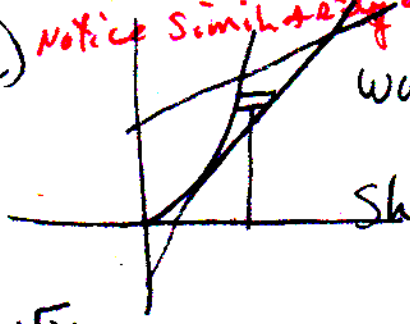


$y=y-2$

$$\pi\int_1^4 \left(\tfrac{1}{2}y+\tfrac{1}{2}\right)^2-\left(\sqrt{y}\right)^2 dy + \pi\int_4^5 \left(\tfrac{1}{2}y+\tfrac{1}{2}\right)^2-(y-2)^2\,dy$$

$$\frac{\pi}{4}\int_1^4 y^2-2y+1-4y\,dy + \frac{\pi}{4}\int_4^5 -3y^2+18y-15\,dy$$

e) Notice similarity of II b ①



Washers: $\pi\int_1^2 (x^2)^2-(2x+1)^2\,dx + \pi\int_2^3 (x+2)^2-(2x-1)^2\,dx$

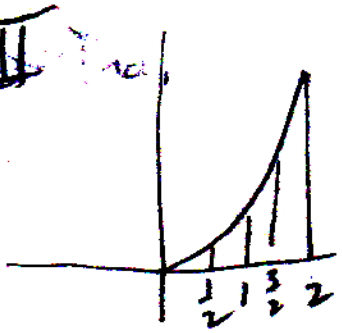
II b (2)
Shells: $2\pi\int_1^4 y\left(\frac{1}{2}(y+1)-\sqrt{y}\right)dy + 2\pi\int_4^5 y\left(\frac{1}{2}y+\frac{1}{2}-(y-2)\right)dy$

$x=\sqrt{y}$
$x=\frac{1}{2}y+\frac{1}{2}$
$x=y-2$

$$2\pi\int_1^4 \tfrac{1}{2}y^2+\tfrac{1}{2}y-y^{\frac{3}{2}}\,dy + 2\pi\int_4^5 \tfrac{1}{2}y^2+\tfrac{1}{2}y-y^2+2y\,dy$$

$$2\pi\int_1^4 \tfrac{1}{2}y^2+\tfrac{1}{2}y-y^{\frac{3}{2}}\,dy + 2\pi\int_4^5 \tfrac{5}{2}y-\tfrac{1}{2}y^2\,dy$$

III



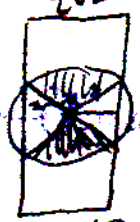
$h=\frac{1}{2}$

$\frac{1}{2}\cdot\frac{1}{3}\left(f(0)+4f(\tfrac{1}{2})+2f(1)+4f(\tfrac{3}{2})+f(2)\right)$

$\frac{1}{6}\left(0+4\cdot\frac{1}{8}+2+4\left(\frac{27}{8}\right)+8\right)$

$\frac{1}{6}\left(10+\frac{1}{2}+\frac{27}{2}\right)$ $\quad \frac{\text{Dep}}{\text{Ind}}=4$

Extra:



$\sqrt{2^2+2^2}=2\sqrt{2}$; right $\frac{\pi}{2}$.

$2\left(\frac{\pi}{2}\cdot 2^2\cdot\frac{1}{2}\right)=2\pi$

area of sector

$+$

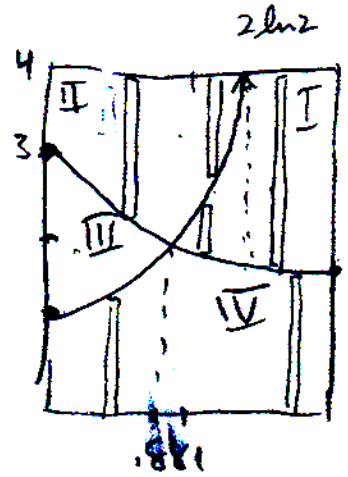
$2\cdot\frac{1}{2}\cdot 2\cdot 2=4=$ area of two rt Δs,

we know the hour right angle because $a^2+b^2=c^2$ true

total "common" area $2\pi+4$.

also 90°

I



$y = e^x$ s.t. $y = 4$

$2\ln 2 = x$

$1.386 = x$

$y = e^{-x}+2$ s.t. $y = e^x$

$e^x = e^{-x}+2$

$e^{2x} - 2e^x - 1 = 0$

$e^x = \frac{2 \pm \sqrt{4+4}}{2}$

$\frac{2 \pm 2\sqrt{2}}{2}$

$e^x = 1 \pm \sqrt{2}$

$x = \ln(1+\sqrt{2})$

$x = .881$

$y = e^{-x}+2 \Big|_{x=2} = \frac{1}{e^2}+2$

$y = 2.135$

areas I, III will be painted with

1.524
.934
2.458

the Gold leaf will cost $2,458.

II $\int_{\ln(1+\sqrt{2})}^{2\ln 2} e^x - (e^{-x}+2)\,dx +$ $\ln(1+\sqrt{2}) = .881$, $2\ln 2 = 1.386$

$\int_{2\ln 2}^{2} 4 - (e^{-x}+2)\,dx$

1.524

II $\int_0^{\ln(1+\sqrt{2})} 4 - (e^{-x}+2)\,dx +$

$\int_{\ln(1+\sqrt{2})}^{2\ln 2} 4 - e^x\,dx = 1.611$

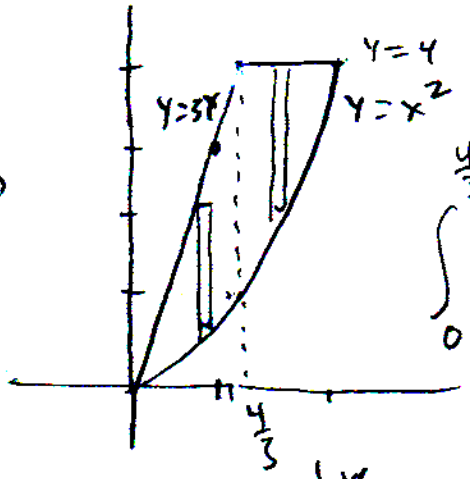
III $\int_0^{\ln(1+\sqrt{2})} e^{-x}+2-e^{-x}\,dx = .934$

IV $\int_0^{\ln(1+\sqrt{2})} e^x\,dx +$

$\int_{\ln(1+\sqrt{2})}^{2} e^{-x}+2\,dx =$

3.930

II



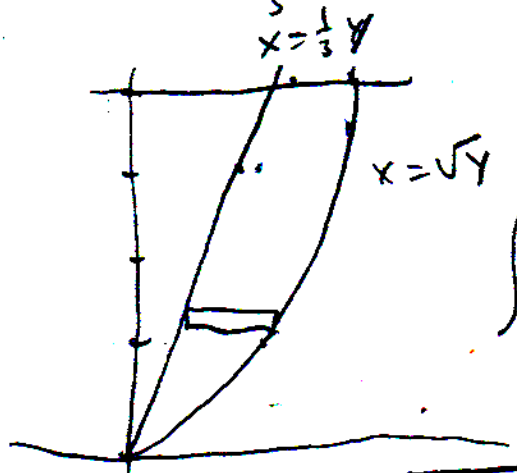
$3x = 4$

$x = \frac{4}{3}$

$\int_0^{4/3} 3x - x^2\,dx +$

$\int_{4/3}^{2} 4 - x^2\,dx$

B)



$\int_0^4 \sqrt{y} - \frac{1}{3}y\,dy$

$2 - \frac{4}{3} = \frac{2}{3} = l_2$

.667

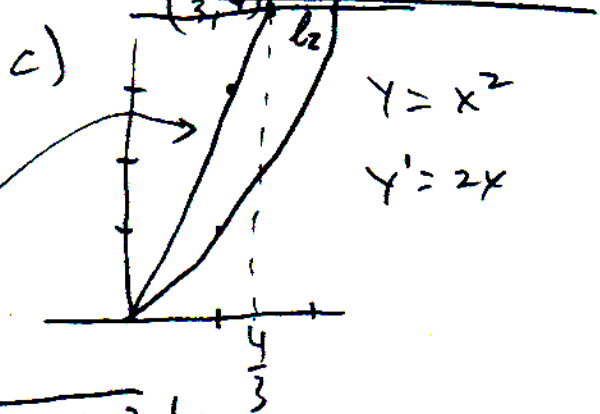
$\sqrt{\left(\frac{4}{3}\right)^2 + 4^2}$

$\frac{4}{3}\sqrt{1+9}$

$\frac{4}{3}\sqrt{10} = l_1$

4.216

C)



$\int_0^2 \sqrt{1+4x^2}\,dx$

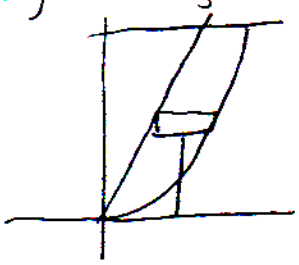
4.647

Total Perimeter

9.530

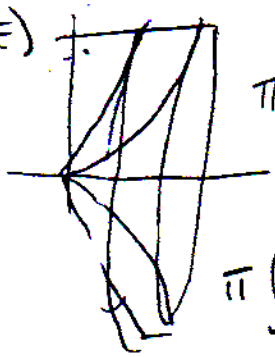
II

D) using II B



$2\pi \int_0^4 y(\sqrt{y} - \frac{1}{3}y)\,dy$

E)



$\pi \int_0^{\frac{y}{3}} (3x)^2 - x^4\,dx + \pi \int_{\frac{y}{3}}^2 16 - x^4\,dx$

III

1) Trapezoidal $\frac{h}{2}(f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n))$

$\frac{10}{2}(0 + 2.38 + 2.50 + 2.62 + 2.60 + 2.55 + 2.51 + 2.30 + 3)$

$5(76 + 100 + 124 + 120 + 110 + 102 + 63) = 3475$

Difference is 75

about 2% error.

Simpson's $\frac{h}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + f(x_n))$

$\frac{10}{3}(0 + 4.38 + 2.50 + 4.62 + 2.60 + 4.55 + 2.51 + 4.30 + 3)$

$\frac{10}{3}(1 + 1 + 152 + 100 + 248 + 120 + 220 + 102 + 123) = 3550$

EXTRA.

II 4 $2.667 = \frac{8}{3}$

13 2.667

C $\frac{4}{3}\sqrt{10} + \frac{2}{3} + \int_0^2 \sqrt{1+4x^2}\,dx$

$4.216 + 0.667 + 4.647$

total 9.530

D) $35.744 = \frac{512\pi}{45}$

E) 35.744

Excellent exam although it doesn't have surface as revolution.

96

25

100	0
90	2
80	4
70	3
60	2
59	6

70.9

This test took about 8–10 min each to grade

Advanced Placement Calculus

Name Calc 98 LAST

Copy original problem.

Per _____ Date Reg Exam

Convince *me* that **you** understand the concept!

While calculators are allowed, a decimal answer without the proper justification has no value.

Last Regular Exam

I A local woodworker, Jake LeSnake, recently was commissioned to produce a very special wooden jewelry box with a decorated lid. The lid will be 2 inches wide by 4 inches high. Jake plans to inlay two curves in the lid using silver wire so that the curves subdivide the lid into four regions. The curves he wishes to use are the graphs of $y = e^x$ and $y = e^{-x} + 2$, where the origin (0, 0) is taken to be the lower left hand corner of the lid. Jake also wants to paint two nonadjacent regions with gold leaf, which costs $1,000 per square inch. Because of your expertise in calculus, Jake wants you to help. Which two regions should Jake paint with gold leaf which will require the least amount of paint and how much will it cost? (of course, you will use three decimal place accuracy for each of the areas.) (40 pts)

II For this section, you are required to ***set up the expression suggested but are NOT required to evaluate that expression.*** The region R, is bounded by the graphs of $y = x^2$ and $y = 3x$, and below the line $y = 4$. (40 pts)

a) Find the area of R using vertical cross sections.

b) Find the area of R using horizontal cross sections.

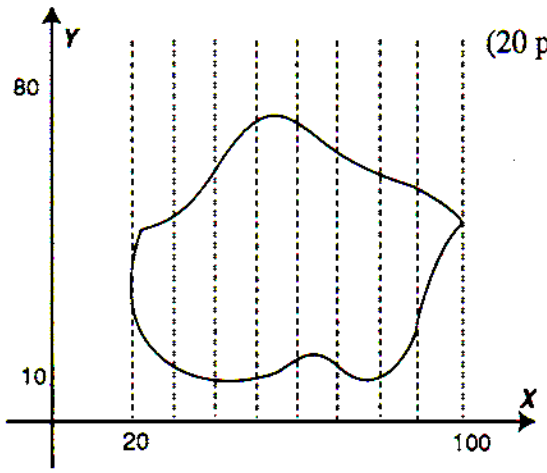
c) Find the perimeter of R.

d) The region R, is rotated about the x-axis. Using shells, find the volume of the resulting solid.

e) The region R, is rotated about the x-axis. Using washers, find the volume of the resulting solid.

III The golf course green is as shown on the right. Jed, the groundsman, prepared the following table to assist you in computing the area of the green. Compare the results of using the trapazoidal method for approximating the integral with using Simpson's rulebased on the following table: (20 pts)

x	20	30	40	50	60	70	80	90	100
width	0	38	50	62	60	55	51	30	3



Extra Credit ---------- 5 pts ----------

Compute the value of the expressions from section II above. Remember that decimal accuracy must be to 3 places if fractional units are used.

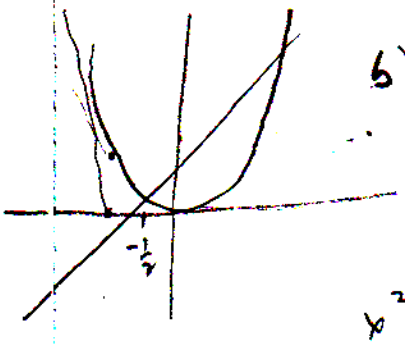
I $y = x+1 \quad y = 2x^2$

a) $2x^2 \overset{\text{Set}}{=} x+1$

5 pts $2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x \in \{-\frac{1}{2}, 1\}$



b) $\int_{-\frac{1}{2}}^{1} x+1-2x^2\,dx$

$\frac{x^2}{2} + x - \frac{2}{3}x^3 \Big|_{-\frac{1}{2}}^{1} =$

$\frac{1}{2} + 1 - \frac{2}{3} - \left(\frac{1}{8} - \frac{1}{2} + \frac{1}{12}\right)$

$\frac{12+24-16-3+12-2}{24}$

$\frac{27}{24} = \frac{9}{8}$ 10 pts

c) 5 pts $y = k(x+1) \overset{\text{Set}}{=} 2x^2$

$2x^2 - kx - k = 0$ has no solutions when $b^2 - 4ac < 0$

$k^2 + 8k = 0$

$k(k+8)$ $\quad k \quad 0 \quad - \quad 0 \quad +$

$-8 \qquad 0$

No solutions if $-8 < k < 0$.

Extra

Since Tangent line Touches Parabola in 1 point the line and the parabola have 1 common point.

if $b^2 - 4ac = 0$

from Ic $k = 0, k = 8$

$\therefore y = 0$ and $y = 8(x+1)$

II

$a(t) = 70 + 12t - 12t^2$

$v(t) = \int a(t)\,dt = \int 70 + 12t - 12t^2\,dt$

$v(t) = 70t + 6t^2 - 4t^3 + c \quad v(0) = 0 \therefore c = 0$

$x(t) = \int v(t)\,dt = \int 70t + 6t^2 - 4t^3\,dt$

$x(t) = 35t^2 + 2t^3 - t^4 + c \quad ; x(0) = 0 \therefore c = 0$

$x(t) = 35t^2 + 2t^3 - t^4$

$-t^2(t^2 - 2t - 35)$

$-t^2(t-7)(t+5)$

b) $x(t) = 0$ when $t = 0, 7$

Particle was at max at $t = 5$, back to origin at $t = 7$. $\therefore$ Time to return, 2 seconds. 10 pts.

$v(t) \overset{\text{Set}}{=} 0$

$-2t(2t^2 - 3t + 35) = 0$

$-2t(2t+7)(t-5)$

$+ \quad 0 \quad - \quad 0 \quad + \quad 0 \quad +$

$-\frac{7}{2} \qquad 0 \qquad 5$

a) max at $t = 5^-$ 1st

$x(5) = -25(-2)(10) = 500$ 10 pts

III

$f'(x) = 4x - 2$ $\quad f(x) = \int f'(x)\,dx$

a) $f(x) = 2x^2 - 2x + C$

$f(1) = 2 - 2 + C = 0$

$C = 0$

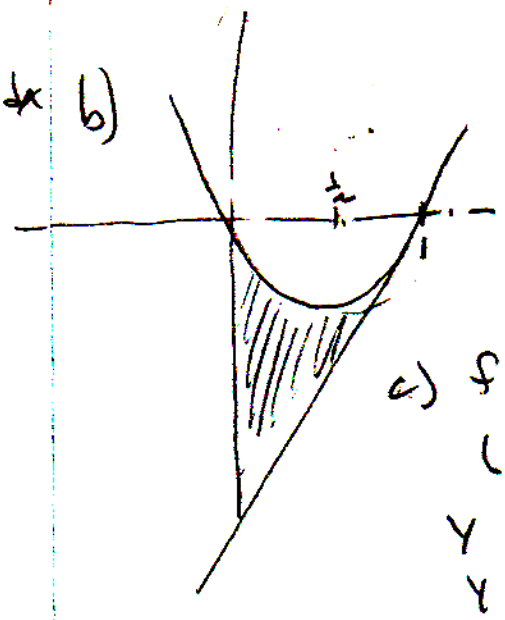
$f(x) = 2x^2 - 2x \quad 2x(x-1)$

$f(0) = 0$, goes through origin

$4x - 2$

$x = \frac{1}{2}$

b)



c) $f'(1) = 4 - 2 = 2 = m$

$(1, 0) \quad m = 2$

$y = 2(x-1)$

$y = 2x - 2$

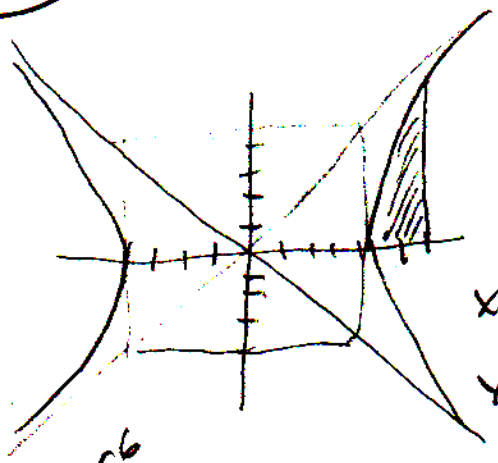
$\int_0^1 2x^2 - 2x - (2x - 2)\,dx$

$\int_0^1 2x^2 - 2x - 2x + 2\,dx$

$\int_0^1 2x^2 - 4x + 2\,dx = \frac{2}{3}x^3 - 2x^2 + 2x \Big|_0^1$

$\frac{2}{3} - 2 + 2 = \frac{2}{3}$

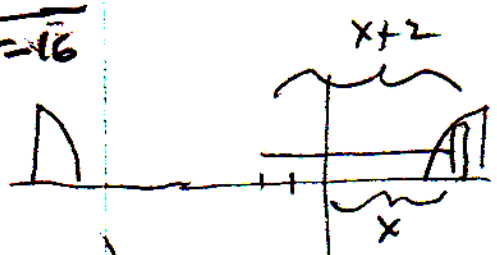
IV



$x^2 - y^2 = 16$

$y = \sqrt{x^2 - 16}$

a) $\int_4^6 \sqrt{x^2-16}\,dx$

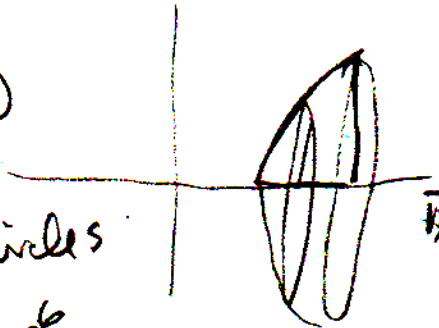


shells.

c) $2\pi \int r h\,dx$

$2\pi \int_4^6 (x+2)\sqrt{x^2-16}\,dx$

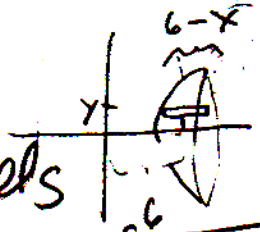
b)



circles

$\pi \int r^2\,dx$

$\pi \int_4^6 (x^2 - 16)\,dx$



shells

$2\pi \int_4^6 \sqrt{x^2-16}\,(6-x)\,dx$

shells in terms of y

$2\pi \int_0^{2\sqrt{5}} y\,(6 - \sqrt{16+y^2})\,dy$

$14 - 0 \quad \frac{x}{y}\,dx$

~~Absence II~~

change It a to extra. Drop It b and current extra. Then this is absolute extra

95
94
93
90

88 - 2
83
81 - 2

~~85~~

81.4

79
76
75 - 2
72
70

62

~~374~~
1302

A.P. Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

Name Calc Ch 10

Per _____ Date 4/29/92

99

Last Regular Exam

I Given a line described: $y = x + 1$ and a parabola described: $y = 2x^2$ (20 pts tot)

a) Find the points of intersection of the line and the parabola.

b) Determine the area enclosed between this line and the given parabola.

c) For what values of k does the line $y = k(x+1)$ have **no** intersection with the parabola?

II A particle moves along the x-axis with acceleration: $a(t) = 70 + 12t - 12t^2$. Time $t > 0$.
If the particle is initially at rest at the origin ... (10 pts ea)

a) find its maximum displacement in the positive direction.

b) find the time taken to return to the origin after the particle was at this maximum displacement.

III For the curve $y = f(x)$ it is given that $f'(x) = 4x - 2$ and the curve passes through the point $(1, 0)$. (10 pts ea)

a) Obtain the equation of this curve and show that it also passes through the origin.

b) Sketch the region bounded by this curve, its tangent at $(1, 0)$ and the axis $x = 0$.

c) Find the equation of this tangent and calculate the area of the region.

IV Let **R** be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 16$, the x-axis and the line $x = 6$. (10 pts ea)

a) Sketch **R**. Set up, ~~but do not integrate, an~~ integral expression in terms of a single variable for the area of **R**, using your graphics calculator and your formulas.

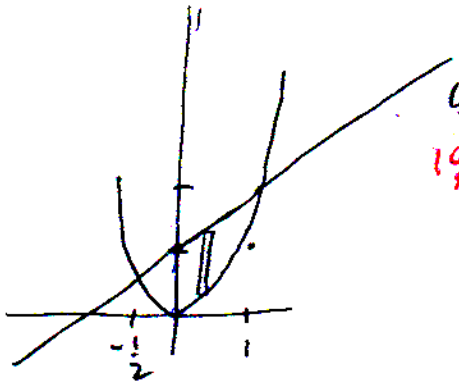
b) In terms of x, ~~but do not integrate,~~ an integral expression in terms of a single variable for the volume of the solid obtained when the region **R** is revolved about the x-axis.

c) In terms of y ..., ~~but do not integrate,~~ a definite integral in terms of a single variable for the volume of the solid generated when **R** is revolved about the line $x = -2$.

Extra Credit — **5 pts**

~~Refering to section~~ I of this exam, find the equations (which are in the form of $y = k(x+1)$) of the **two** tangents to the parabola which pass through the point $(-1, 0)$.

I $y = x+1 \quad y = 2x^2$



a) $x+1 \overset{\text{Set}}{=} 2x^2$ — 3 pts

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = \{-\frac{1}{2}, 1\}$

$(-\frac{1}{2}, \frac{1}{2}) \quad (1, 2)$

b) $\int_{-\frac{1}{2}}^{1} x+1-2x^2\,dx$ — 10 pts

$\frac{x^2}{2} + x - \frac{2}{3}x^3 \Big|_{-\frac{1}{2}}^{1} =$

$\frac{1}{2} + 1 - \frac{2}{3} - \left(\frac{1}{8} - \frac{1}{2} + \frac{1}{12}\right)$

$\frac{1}{2} + 1 - \frac{2}{3} - \frac{1}{8} + \frac{1}{2} - \frac{1}{12}$

$2 \quad \frac{-16-3-2}{24}$

$2 - \frac{21}{24} = \frac{27}{24} = \frac{9}{8}$

c) $y = k(x+1) \overset{\text{Set}}{=} 2x^2$ — 12 pts

$2x^2 - kx - k = 0$

This has no solutions when $b^2 - 4ac < 0$

$k^2 + 8k = 0$

$k(k+8) = 0$

$+ \;0\; - \;0\; +$

$-8 \qquad 0$

No solution if $-8 < k < 0$

II

a) $f'(x) = 4x - 2 \qquad (1,0)$

$\frac{dy}{dx} = 4x - 2$

$\int dy = \int 4x - 2\,dx$

$y = 2x^2 - 2x + c \overset{\text{Set}}{=} 0$

$f(1) = 2 - 2 + c = 0$

$c = 0$

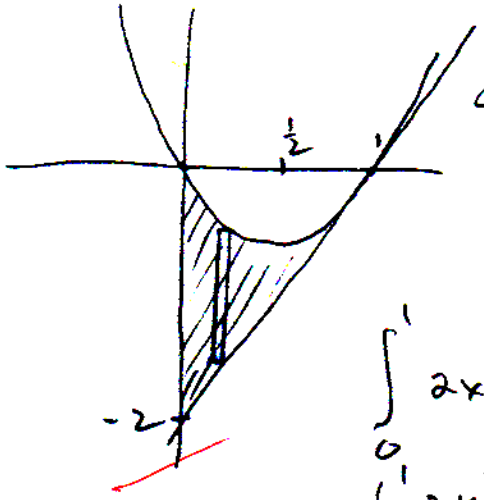
$f(x) = 2x^2 - 2x$

$f(0) = 0 - 0$

$= 0$

So f also passes through origin.

II b)



c) $f'(1) = 4 - 2 = 2$

$(1,0)$

$y - 0 = 2(x-1)$

$y = 2x - 2$

$\int_0^1 2x^2 - 2x - (2x-2)\,dx$

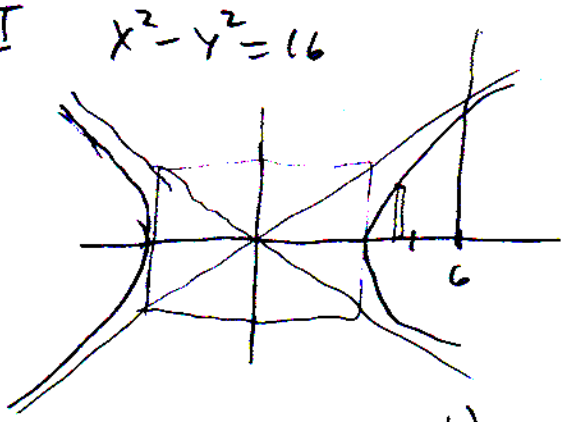
$\int_0^1 2x^2 - 4x + 2\,dx$

$\frac{2}{3}x^3 - 2x^2 + 2x \Big|_0^1 =$

$\frac{2}{3} - 2 + 2 = \frac{2}{3}.$

This value minus this value } -3

III $x^2 - y^2 = 16$



a) $\int_4^6 \sqrt{x^2-16}\,dx \approx 5.717$

$y = \sqrt{x^2-16}$

$dy = \frac{x}{\sqrt{x^2-16}}$

$\pi \int r^2\,dx$ "Disks"

b) $\pi \int_4^6 (x^2-16)\,dx = 58.643$

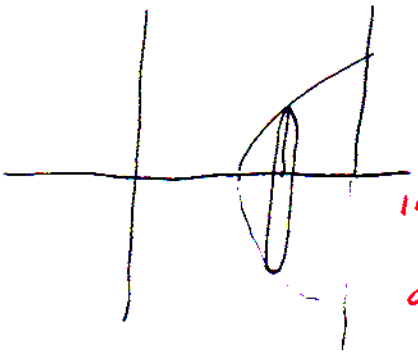
$2\pi \int_{2\sqrt{5}}^{6} \text{radius} \cdot C(x)$

$2\pi \int_0^{} (6 - x)\,dy$

Now must convert to x.

$2\pi \int_4^6 \sqrt{x^2-16}\,(6-x)\,\frac{x\,dx}{\sqrt{x^2-16}}$

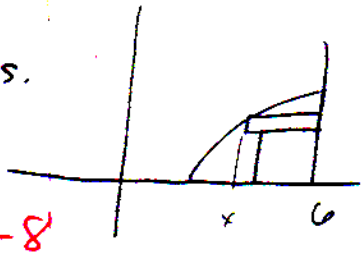
$2\pi \int_4^6 6x - x^2\,dx = 58.643$



in shells.

IN wrong variable and no attempt to convert -8

ANS correct numerical answer



$6^2 - y^2 = 16$

$y^2 = 20$

$y = 2\sqrt{5}$

$x^2 = y^2 + 16$

$x > \sqrt{y^2+16}$

Washers

$\pi \int_0^{2\sqrt{5}} 8^2 - \left(\sqrt{y^2+16} + 2\right)^2 dy$

$\pi(82.497)$

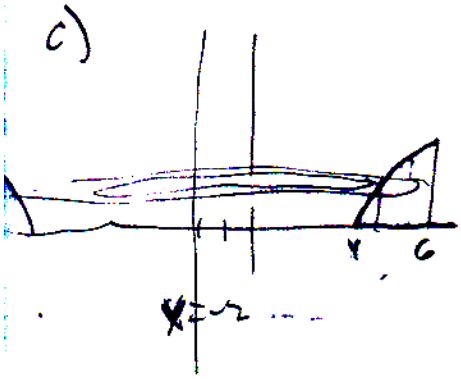
259.171

IN x with shells

$2\pi \int_4^6 (x+2)\sqrt{x^2-16}\,dx$

259.171

c)



Extra.

$a(t) = 70 + 12t - 12t^2$

$v(t) = 70t + 6t^2 - 4t^3 + C$

$v(0) = C \overset{\text{set}}{=} 0 \quad \therefore C = 0$

$x(t) = 35t^2 + 2t^3 - t^4 + C_2 \quad \therefore C_2 = 0$

$x(t) = 35t^2 + 2t^3 - t^4$

$-t^2(t^2 - 2t - 35)$

$-t^2(t-7)(t+5)$

$v(t) = -2t(2t^2 - 3t - 35)$

$-2t(2t+7)(t-5)$

$t = \{0, -\frac{7}{2}, 5\}$

$t = 5$ others are reject.

$a(5) = 70 + 60 - 12 \cdot 25 < 0$

$\therefore$ max f "test"

$x(5) = -25(-2)(10)$

$= 500$

Very easy Last exam.
Probably should have added Surface area question to part III
None. They barely finished this.
apparently reading is not a trait shared by this group.

100 ↑	0
90 ↑	0
80	4
70	5
60	2
59	2

$\frac{941}{13} = 72.4$

88
↓
46

Advanced Placement Calculus Name Calc 10

Copy original problem. Per _____ Date 99

Convince *me* that **you** understand the concept!

Last Regular Exam

I Given a line described: $y = x + 1$ and a parabola described: $y = 2x^2$. (25 pts tot)

a) Find the points of intersection of the line and the parabola.

b) Determine the area trapped between the given line and the given parabola.

c) For what values of k does the line $y = k(x + 1)$ have **no** intersection with the parabola?

II For the curve $y = f(x)$ it is given that $f'(x) = 4x - 2$ and the curve passes through the point $(1, 0)$. (10 pts ea)

a) Obtain the equation of this curve and show that it also passes through the origin.

b) Sketch the region bounded by this curve, its tangent at $(1, 0)$ and the axis $x = 0$.

c) Find the equation of this tangent and calculate the area of the region.

III Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 16$, the x-axis and the line $x = 6$. (15 pts ea)

a) Sketch R. Set up the integral expression in terms of a single variable for the area of R. Using your grapher, evaluate your formula.

b) Set up an integral expression in terms of x for the volume of the solid obtained when the region R is revolved about the x-axis. Using your grapher, evaluate your formula.

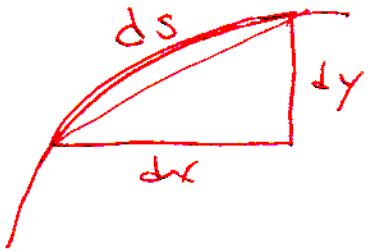
c) Set up a definite integral in terms of a y for the volume of the solid generated when R is revolved about the line $x = -2$. Using your grapher, evaluate your formula.

Extra Credit ---------- 5 pts ----------

A particle is initially at rest at the origin.

A particle moves along the x axis with acceleration $a(t) = 70 + 12t - 12t^2$ where time t is positive. Find the particle's maximum displacement in the positive direction.

Pg 306



$ds \approx$ slant height

$\Delta s \approx$ slant height.

$\Delta s \approx 2\pi y \Delta x$ ← These CANNOT Both be right.

$\Delta s \approx 2\pi y \sqrt{(\Delta x)^2 + (\Delta y)^2}$ ←

This is what we found the "Actual" value to be.

This is what we are trying to use in my demo.

Concept error.

$S = \int 2\pi R \, ds$ (Pg 417) Pg ~~407~~ 410

Pg 416 417

ds definition

(159) Given $\int \frac{e^x}{x}\,dx$ is not elementary

WTP $\int \frac{1}{\ln x}\,dx$ is Not elementary.

$\int \frac{e^x}{x}\,dx$ Let $u = e^x$ $v = \ln x$

$du = e^x dx$ $dv = \frac{1}{x}\,dx$

$e^x \ln x - \int e^x \ln x\,dx$

Let $u = \ln x$ $v = e^x$

$du = \frac{1}{x}\,dx$ $dv = e^x dx$

$= e^x \ln x + \int \frac{e^x}{x}\,dx$

Let $u = \ln x$
$e^u = x$
$e^u du = dx$
now substitute into $\int \frac{dx}{\ln x}$
etc

Just a circle.

$(e^x \ln x) + \frac{e^y}{x}$

Let $e^x = \ln u$

$x = \ln(\ln u)$

$dx = \frac{1}{u \ln u}\,du$

$\int \frac{e^x}{x}\,dx$

$\int \frac{\ln u}{\ln \ln(u)} \frac{1}{u \ln u}\,du = \int \frac{du}{u \ln \ln(u)}$

Let $e^x = u$

$x = \ln u$

$dx = \frac{1}{u}\,du$

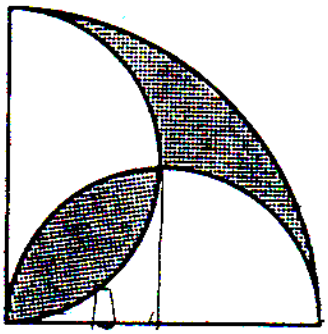
Finally Solution:

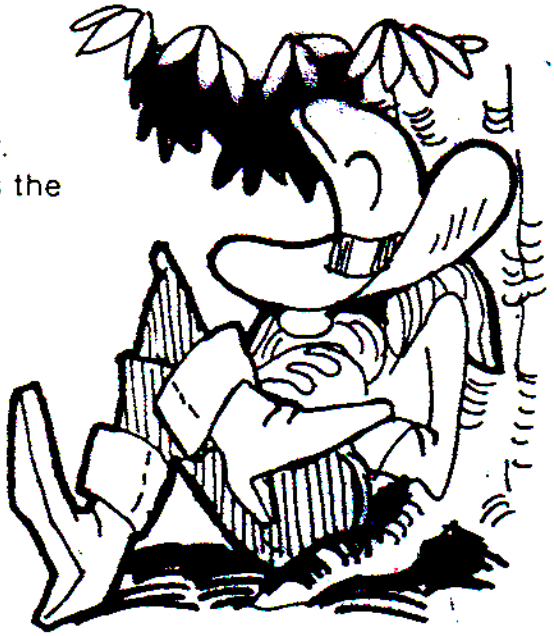
$\int \frac{e^x}{x}\,dx = \int \frac{u}{\ln u} \cdot \frac{1}{u}\,du = \int \frac{du}{\ln u}$

If $\int \frac{e^x}{x}\,dx$ is Non elementary Then So

IN THE SHADE

Inside this quarter-circle are two semicircles having the same radius, r. Which of the two shaded regions has the greater area?





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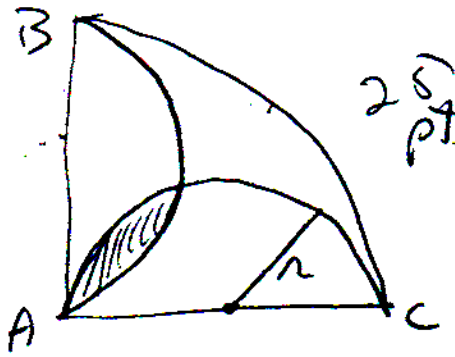
Inside this quarter circle are two semicircles having the same radius r.



25 pts

Let the area common to all 3 circle portions be R.

Given: $\int \sqrt{2ax-x^2}\,dx = \frac{1}{2}\left[(x-a)\sqrt{2ax-x^2} + a^2 \sin^{-1}\left(\frac{x-a}{|a|}\right)\right]$

4 pts

Compute the area of R.

arc AC $(x-r)^2+y^2=r^2$

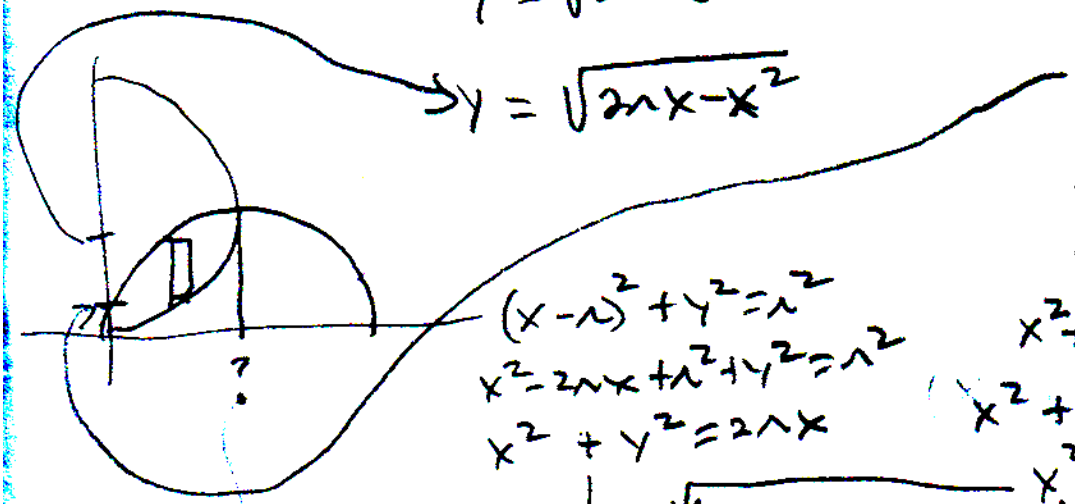
$y=\sqrt{r^2-(x-r)^2}$

$y=\sqrt{2rx-x^2}$

arc AB $x^2+(y-r)^2=r^2$

$y-r=\pm\sqrt{r^2-x^2}$

$y=r-\sqrt{r^2-x^2}$ ← note we want the bottom of the circle.



$(x-r)^2+y^2=r^2$

$x^2-2rx+r^2+y^2=r^2$

$x^2+y^2=2rx$

$x^2+(y-r)^2=r^2$

$x^2+y^2-2ry+r^2=r^2$

$x^2+y^2=2ry$

$2rx=2ry$

$x=y$

$2x^2=2rx$

$2x(x-r)=0$

$x=r$.

$$\int_0^r \sqrt{2rx-x^2} - \left(r-\sqrt{r^2-x^2}\right)dx$$

$$\int_0^r \sqrt{2rx-x^2}\,dx - \int_0^r r\,dx + \int_0^r \sqrt{r^2-x^2}\,dx$$

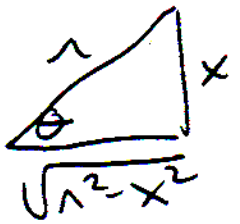
$$\int_0^{\hat{}} \sqrt{2r-x^2}\,dx = \frac{1}{2}\left[(x-r)\sqrt{2rx-x^2} + r^2 \sin^{-1}\left(\frac{x-r}{|r|}\right)\right]_0^{R}$$

$$= \frac{1}{2}\left[0 + r^2 \cdot 0 - \left[-r\cdot 0 + r^2 \sin^{-1}(-1)\right]\right]$$

$$r^2\left(-\frac{\pi}{2}\right)$$

$$= \frac{1}{2}\cdot\frac{\pi r^2}{2} = \frac{\pi r^2}{4}$$

$$\int_0^r \sqrt{r^2-x^2}\,dx$$



$\sin\theta = \frac{x}{r}$ $\quad x=0\ \theta=0$ $\quad x=r\ \theta=\frac{\pi}{2}$

$r\sin\theta = x$

$r\cos\theta\,d\theta = dx$

$$\int_0^{\frac{\pi}{2}} r^2\cos^2\theta\,d\theta = \frac{1}{2}r^2\int_0^{\frac{\pi}{2}}(1+\cos 2\theta)\,d\theta$$

$$= \frac{1}{2}r^2\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}}$$

$$= \frac{r^2}{2}\left[\frac{\pi}{2} + \frac{1}{2}\cdot 0 - 0\right]$$

$$= \frac{\pi r^2}{4}$$

$$\int_0^{r} \sqrt{2rx-x^2}\,dx - \int_0^r x\,dx + \int_0^r \sqrt{r^2-x^2}\,dx =$$

$$\frac{\pi r^2}{4} - r^2 + \frac{\pi r^2}{4}$$

$$\frac{\pi r^2}{2} - r^2$$

$$\text{area} = \frac{r^2}{2}(\pi-2)$$

Calculus Ch 10 **"OBVIOUS", as applied to Calculus** Mr. Mumaugh

Pure Enjoyment # 2 Name _____

This "honor quiz" is to be done by you ***totally* on your own**. You may use your text, notes and calculator. You are not allowed to discuss your solutions with anyone until the quiz has been turned in at the date and time shown on the homework schedule. Answer the questions on your own paper and staple your work to this sheet.

I Given $x = t^2 + 5$, $y = t^3 - 9t$, for $-4 \le t \le 4$.

a) Sketch the curve with a window of $x \varepsilon [0, 20]$ and $y \varepsilon [-20, 20]$ T-Step = .2.

b) Determine the values of t and x when $y = 0$.

c) Find the slopes of the tangent lines at each of the **three** values of t found in part (b).

d) Using an integral in terms of t, find all points when the tangent line is horizontal.

e) Find the area enclosed by the loop. (Hint: explore the curve using the TRACE command before attempting this problem.

f) Sames as part "e" above except ... "in terms of x" etc.

II We use $x = r\cos\theta$ and $y = r\sin\theta$ as a means to convert back and forth between cartesian and polar coordianates. Given: $r = a\sin\theta + b\cos\theta$.

a) Use the given and rewrite x and y so the variable r is eliminated.

b) Using the results of part a, find $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}}$.

Do not use a calculator to determine this.

c) Convert r to cartesian form.

d) Determine the cartesian coordinates of the center of the circle and specifically state the radius.

e) Using the results of part c, find $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}}$.

Do not use a calculator to determine this.

f) Copy and fill in the table on the right:

	θ	r
1	0	b
2	$\frac{\pi}{6}$	$\frac{a}{2} + b\frac{\sqrt{3}}{2}$
3	$\frac{\pi}{4}$	$a\frac{\sqrt{2}}{2} + b\frac{\sqrt{2}}{2}$
4	$\frac{\pi}{3}$	
5	$\frac{\pi}{2}$	
6	$\frac{2\pi}{3}$	
7	$\frac{3\pi}{4}$	
8	$\frac{5\pi}{6}$	
9	π	$-b$
10	$\frac{7\pi}{6}$	$-\frac{a}{2} - b\frac{\sqrt{3}}{2}$

g) What is the significance of line 10 (as compared to line 2), for example?

h) Find the area bounded by the graph of r. Use the formula for the area bounded by the graph of a polar function. (Hint: The information gained in part g is vital to this problem. Use it wisely.)

i) Using the formula for the length of a piece of the graph of a polar function, find the circumference of the figure.

j) Using IIc and appropriate cartesian formulas, verify your answers to IIh and IIi.

The work on this problem was done by me with no assistance from another person _____
your signature

Calculus Ch 10 **How much power is *your* vote worth?** J. Mumaugh

(aka .... Who put the DUH in Florida?)

a couple of definitions ...

We define $f_i(p)$ as the probability that voter i will vote for a particular bill which has "acceptability" p.

We define "voting power" as the probability that voter i's vote will make a difference on a single bill *averaged for all possible probabilties.*

an example ...

The situation: It takes 3 votes to pass the bill. Voter "A" has 2 votes. Voters "B" and "C" each have one vote. This is summarized as: {3;2,1,1}

How much power do voters A, B, and C have?

The probability that Voter A's vote makes a difference is if Voter B votes Yes and Voter C votes No or if Voter A votes No and Voter B votes Yes or If Voter B votes Yes and Voter C votes Yes. This is summarized as:

$$\frac{1}{1-0}\int_0^1 2p-p^2\,dp = p^2-\frac{1}{3}p^3\Big|_0^1 = \frac{2}{3}\quad f_A(p) = p(1-p)+(1-p)p+p\cdot p$$

Notice that if Voter B votes no and Voter C votes no, then Voter A's vote has no meaning and is therefore not included in this list.

Now we integrate for all possibilities ... $\frac{1}{1-0}\int_0^1 2p-p^2\,dp = p^2-\frac{1}{3}p^3\Big|_0^1 = \frac{2}{3}$

Conclusion: Voter A has 66.7% of the power in this situation while Voter A has 50% of the vote! Does that make Voter A a dictator? a king? Does Voter A have veto power? ....

Given the following voting situations ....

1 {6;7,1,1,1}

2 {6;3,3,3,1}

3 {7;4,3,2,1}

4 {6;4,3,2,1}

5 {5;4,3,2,1}

6 {4;4,3,2,1}

7 {3;4,3,2,1}

8 {2;4,3,2,1}

Describe the power for each voter in each situation. Include all computations and explanations in support of your conclusions